Name: Enrolment No:	

UPES

End Semester Examination, May 2024

Course: Integral Equations & Calculus of Variations — **Semester: VIII**
Program: B.Sc. Physics — **Time: 03 hrs.**
Course Code: MATH4019 — **Max. Marks: 100**

Instructions: Read all the below mentioned instructions carefully and follow them strictly:
1) Mention Roll No. at the top of the question paper.
2) ATTEMPT ALL THE PARTS OF A QUESTION AT ONE PLACE ONLY.

SECTION A
(5Qx4M=20Marks)

S. No.		**Marks**	**CO**
Q 1	Convert the following initial value problem into an integral equation: $\frac{d^2y}{dx^2} + A(x)\frac{dy}{dx} + B(x)y = f(x)$ with $y(a) = y_0, y'(a) = y'_0$.	**4**	**CO1**
Q 2	Show that the initial value problem corresponding to the Volterra integral equation $y(x) = 1 + \int_0^x y(t)dt$ is $\frac{dy}{dx} - y = 0, y(0) = 1$.	**4**	**CO1**
Q 3	State Hilber Schimdt theorem.	**4**	**CO1**
Q 4	Prove that if the kernel of an integral equation is symmetric then all its iterated kernels are also symmetric.	**4**	**CO2**
Q 5	Find the extremal of the functional $\int_1^3 y(3x - y)dx$ that satisfy the boundary conditions $y(3) = \frac{9}{2}, y(1) = 1$.	**4**	**CO3**

SECTION B
(4Qx10M= 40 Marks)

Q 6	Find the solution of the following integral equation $y(x) = x^2 + 1 + \frac{3}{2}\int_{-1}^{1}(xt + x^2t^2)y(t)dt$ using Hilbert-Schmidt theorem,	**10**	**CO2**
Q 7	Suppose a particle is sliding from a point $(x_1,\ y_1)$ to another point $(x_2,\ y_2)$, if its rate of motion $v\left(= \frac{dx}{dt}\right)$ is equal to x. Find the path on which the particle will take minimum time.	**10**	**CO3**
Q 8	Prove that the shortest distance between two fixed points in a plane is straight line.	**10**	**CO4**

Q 9	Prove that the resolvent kernel $R(x,t,\lambda)$satisfies the integral equation $R(x,t,\lambda) = K(x,t) + \lambda \int_a^b K(x,z)R(z,t,\lambda)dz$ where the Fredholm integral equation is given by $y(x) = f(x) + \lambda \int_a^b K(x,t)y(t)dt,$ **OR** Find the eigenvalues and eigenfunctions of the homogeneous Fredholm integral equation of second kind: $y(x) = \lambda \int_0^1 (2xt - x^2)y(t)dt.$	**10**	**CO2**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	Find the extremal of the functional $I[y(x)] = \int_0^1 (1 + y''^2)dx$ that satisfy the conditions $y(0) = 0, y'(0) = 1, y(1) = 1, y'(1) = 1.$	**20**	**CO3**
Q 11	Find the shortest distance between the circle $x^2 + y^2 = 4$ and a straight line $2x + y = 6.$ **OR** Find the shortest distance between the parabola $y = x^2$ and the straight line $y = x - 5.$	**20**	**CO4**