Name: Enrolment No:	

UPES

End Semester Examination, May 2024

Programme Name: Integrated B.Sc.-M.Sc. Mathematics **Semester : VI**
Course Name : Theory of Partial Differential Equations **Time : 03 hrs**
Course Code : MATH 3050 **Max. Marks: 100**
Nos. of page(s) : 02

Instructions: Attempt all questions.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q1	Form a partial differential equation corresponding to the family of surfaces given by $z = f(x^2 - y^2)$, where f is an arbitrary function.	**4**	**CO1**
Q2	Prove that the characteristic curves on xy −plane for the PDE $$x^2u_{xx} - y^2u_{yy} = x^2y^2 + x; x > 0$$ are straight lines through the origin.	**4**	**CO1**
Q3	Describe the region in the lower half plane where the second order PDE: $$y^3u_{xx} - (x^2 - 1)u_{yy} = 0$$ is hyperbolic.	**4**	**CO2**
Q4	Find the D-Alembert's solution of the initial value problem: $u_{tt} = 4u_{xx}, t > 0, -\infty < x < \infty$ satisfying the conditions $u(x,0) = x, u_t(x,0) = 0$.	**4**	**CO2**
Q5	Use direct integration to find the value $u(1,1)$ if $\frac{\partial u}{\partial x} = xe^x$ such that $u(0,y) = y$.	**4**	**CO4**

SECTION B
(4Qx10M= 40 Marks)

S. No.		Marks	CO
Q 6	Solve the Cauchy problem $\frac{\partial u}{\partial x} - 2\frac{\partial u}{\partial y} = u - 1$ subject to the Cauchy data $u(ky, y) = 2y$ giving reasons for any restriction that must be placed on k.	**10**	**CO1**
Q7	Find the solution of the IVP $u_t + 2u_x = 0$ with $u_t(x,0) = x$. Prove that the solution is not unique.	**10**	**CO2**
Q8	Find the solution of Dirichlet problem on the square $0 \leq x \leq 1, 0 \leq y \leq 1$: $$u_{xx} + u_{yy} = 0$$	**10**	**CO3**

	with $u(0,y) = u(1,y) = 0$ for $0 \le y \le 1$ and $u(x,0) = 0$ and $u(x,1) = \cosh n\pi x$ for $0 \le x \le 1$		
Q9	Consider the heat equation defined on $x > 0$; $$u_t = u_{xx} \text{ for } x > 0, t > 0$$ with $u(0,t) = 0$ for $t > 0$ and $u(x,0) = \begin{cases} 1 & \text{for } 0 \le x \le h \\ 0 & \text{for } x > h \end{cases}$ Find the solution $u(x,t)$. **OR** Derive the most general solution of wave equation on real axis: $$\frac{1}{c^2} u_{tt} = u_{xx}, -\infty < x < \infty, t > 0$$ such that $u(x,0) = f(x)$ and $\frac{\partial u}{\partial x}(x,0) = g(x)$	**10**	**CO3**
SECTION-C **(2Qx20M=40 Marks)**			
Q10	Consider the wave equation with a forcing term $F(x) = \cos x$ as follows: $$\frac{\partial^2 y}{\partial t^2} = \frac{\partial^2 y}{\partial x^2} + F(x) \quad \text{for } 0 < x < 2\pi, t > 0$$ $$y(0,t) = y(2\pi,t) = 0 \text{ for } t \ge 0;\ y(x,0) = \frac{\partial y}{\partial t}(x,0) = 0 \text{ for } 0 \le x \le 2\pi$$ Using suitable transformation reduce it into homogeneous wave equation and hence find the solution $y(x,t)$.	**20**	**CO2**
Q11	Solve the heat conduction problem: $$\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2} \text{ for } 0 \le x \le \pi, t > 0$$ $\frac{\partial u}{dx}(0,t) = 0, u(\pi,t) = 100^{\circ}C$ and $u(0,t) = 50^{\circ}C$ **OR** Consider a thin rod of length 2π in which the temperature distribution $u(x,t)$ is governed by the equation $\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}$. Find the temperature distribution function $u(x,t)$ if the left end of rod is perfectly insulated and the right end is at zero temperature with the initial distribution given by $$u(x,0) = \begin{cases} x, & \text{if } 0 \le x \le \pi \\ 2\pi - x, & \text{if } \pi < x \le 2\pi \end{cases}$$	**20**	**CO4**