Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UPES

End Semester Examination, May 2024

Course: Riemann Integration and Series of Functions **Semester: VI**

Program: B.Sc. (H) Mathematics **Time : 03 hrs.**

Course Code: MATH 3044 P **Max. Marks: 100**

Instructions: There are total 11 questions. Answer the questions in legible handwriting mentioning solutions to question number properly

SECTION A

(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Justify with an example that a real function can be Riemann integrable over an interval but not continuous therein.	**4**	**CO1**
Q 2	If n is a positive integer, show that $$2^n\Gamma\left(n+\frac{1}{2}\right)=1.3.5\ldots\ldots(2n-1)\sqrt{\pi}.$$	**4**	**CO2**
Q 3	Define (i) no where convergent power series, and (ii) everywhere convergent power series.	**4**	**CO3**
Q 4	Find the radius of convergence for the series of the functions given by $\sum_{n=0}^{\infty}\frac{x^n}{n!}$.	**4**	**CO3**
Q 5	For the sequence of functions $f_n(x)=\frac{x^2}{x^2+n}$ on the interval $[0,\infty)$, determine if it converges pointwise and/or uniformly.	**4**	**CO4**

SECTION B

(4Qx10M= 40 Marks)

There is an internal choice in Q9

S. No.		Marks	CO
Q 6	Evaluate $\int_0^3 \frac{1}{(x-1)^{\frac{2}{3}}}dx$. Show that the Beta function is symmetric in positive real variables x and y.	**10**	**CO2**
Q 7	For $\int_0^{\pi} tanx\,dx$, discuss the convergence of near the singularity at $x=\frac{\pi}{2}$.	**10**	**CO3**
Q 8	State Leibniz's rule for differentiation under the integral sign. Apply Leibniz's rule to compute the derivative of the integral $G(x)=\int_0^x \cos(tx)dt$ with respect to x.	**10**	**CO4**
Q 9	(i) Define a metric space citing a proper example. Find all metrics on a set X consisting of one point, and consisting of two points. Does $d(x,y)=(x-y)^2$ define a metric on the set of real numbers?	**10**	**CO4**

	OR (ii) Define a metric space citing a proper example. Let d be a metric on X. Determine all constants k such that (a) kd, and (b) $d + k$ is a metric on X.		
SECTION-C **(2Qx20M=40 Marks)** **There is an internal choice in Q11**			
Q 10	Investigate the convergence of the Beta function for varying parameter values α and β. Determine if there are any specific ranges of values for which the function converges more rapidly and slowly.	**20**	**CO3**
Q 11	(i) Show that the sequence $< f_n = \frac{n^2 x}{1+n^4 x^2} >$ does not converge uniformly on [0, 1]. **OR** (ii) Define the uniform norm. Using the uniform norm show that the sequence $G_n(y) = (\frac{y}{2})^n(1-y)$ for $y \in A = [0,1]$ converges uniformly to $G(y) = 0$. State Cauchy criterion for Uniform convergence.	**20**	**CO4**