Name:

Enrolment No:

UPES

End Semester Examination, May 2024

Course: Numerical Methods **Semester: IV**

Program: B.Sc. (Hons.) Chem./B.Sc. (Hons.) Geology/B.Sc. (Hons.) Physics **Time : 03 hrs.**

Course Code: MATH2017G **Max. Marks: 100**

Instructions: Attempt all questions

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Prove that $$\Delta \ln f(x) = \ln\left\{1 + \frac{\Delta f(x)}{f(x)}\right\},$$ where Δ is the forward difference operator.	**4**	**CO1**
Q 2	Evaluate $\sqrt{12}$ to four decimal places by Newton-Raphson method.	**4**	**CO2**
Q 3	Construct the backward difference table for the data: x: 2, 4, 6, 8 $f(x)$: 14.5, 16.3, 17.5, 18	**4**	**CO3**
Q 4	Show that the Trapezoidal rule is exact for polynomials of degree less than or equal to one, and it is not exact for polynomials of degree two.	**4**	**CO4**
Q 5	Using Picard's method, obtain the 2nd approximation, if $\frac{dy}{dx} = 1 + xy$ with $y(0) = 2$.	**4**	**CO6**

SECTION B
(4Qx10M= 40 Marks)

Q 6	Using Newton-Raphson method, find the real root of $f(x) = x\sin x + \cos x$, which is near $x = \pi$ correct to three decimal places.							**10**	**CO2**
Q 7	Given the following table: x: 1, 2, 3, 4, 5, 6, 7, 8 $f(x)$: 1, 8, 27, 64, 125, 216, 343, 512 Construct the difference table and compute $f(1.5)$ and $f(7.5)$.							**10**	**CO3**
Q 8	Evaluate $\int_0^1 \frac{dx}{1+x}$ by dividing the interval into 8 equal parts using Simpson's rule. Hence evaluate $\log_e 2$ approximately.							**10**	**CO4**

<table>
<tr><td>Q 9</td><td>Solve equations:

$$27x + 6y - z = 85$$
$$x + y + 54z = 110$$
$$6x + 15y + 2z = 72$$
using Gauss-Seidel method. Use only four iterations.

OR

Calculate the solution of the system of equations:

$$20x - y + z = 23.28$$
$$x + 15y - z = 29.92$$
$$2x + y - 20z = -55.64$$
using Gauss-Jacobi method correct to three decimal places.</td><td>10</td><td>CO5</td></tr>
<tr><td colspan="4">SECTION-C
(2Qx20M=40 Marks)</td></tr>
<tr><td>Q 10</td><td>A. Evaluate $f'(1.1)$ and $f"(1.1)$, from the following table:
<table><tr><td>x</td><td>1.1</td><td>1.2</td><td>1.3</td><td>1.4</td><td>1.5</td></tr><tr><td>$f(x)$</td><td>2.0091</td><td>2.0333</td><td>2.0692</td><td>2.1143</td><td>2.1667</td></tr></table>B. Evaluate $\int_0^1 (4x - 3x^2)dx$, taking 10 intervals, by Trapezoidal rule. Compute the exact value and find the absolute and relative errors in your result.</td><td>10+10</td><td>CO4</td></tr>
<tr><td>Q 11</td><td>Compute $y(0.8)$, by fourth order Runge-Kutta method correct to five decimal places, from the equation:
$$\frac{dy}{dx} = xy, \quad y(0) = 2,$$
taking step-length $= 0.2$
OR
Using Milne's predictor-corrector method, find $y(0.5)$ for the initial value problem $\frac{dy}{dx} = 2e^x - y, \quad y(0) = 2,$ with $h = 0.1$
Calculate all the required initial values by Euler's method correct to three decimal places.</td><td>20</td><td>CO6</td></tr>
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