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| **Name:**<br>**Enrolment No:** | UPES UNIVERSITY OF TOMORROW |

**UPES**
**End Semester Examination, May 2024**

**Course: Analytical Geometry** **Semester: II**
**Program: B. Sc. (Mathematics by Research)** **Time: 03 hrs.**
**Course Code: MATH 1069** **Max. Marks: 100**

**Instructions: Attempt all questions.**

**SECTION A**
**(5Qx4M=20Marks)**

| S. No. | | Marks | CO |
|---|---|---|---|
| Q 1 | Find the equation of the circle cutting off intercepts 4 and 6 on the coordinate axes and passing through the origin. | **4** | **CO1** |
| Q 2 | Show that the locus of the point of intersection of perpendicular tangents to a parabola is the directrix. | **4** | **CO2** |
| Q 3 | The foot of the perpendicular from the origin to a plane is (13, -4, -3). Find the equation of the plane. | **4** | **CO4** |
| Q 4 | Find the equation of the hyperbola whose centre is (1, 0), one focus is (6, 0) and length of transverse axis is 6. | **4** | **CO3** |
| Q 5 | The equation $25(x^2-6x+9)+16y^2=400$ represents an ellipse. Find the centre and foci of the ellipse. | **4** | **CO1** |

**SECTION B**
**(4Qx10M= 40 Marks)**

| S. No. | | Marks | CO |
|---|---|---|---|
| Q 6 | Prove that the angle between the lines given by $x+y+z=0$, $ayz+bzx+cxy=0$ is $\frac{\pi}{2}$ if $a+b+c=0$. | **10** | **CO3** |
| Q 7 | A circle of radius 2 and the centre (2, 3, 0) lies in the plane $z=0$. Find the equation of the sphere containing this circle and passing through the point (1, 1, 1). | **10** | **CO2** |
| Q 8 | The equation of two diameters of a circle are $2x+y-3=0$ and $x-3y+2=0$. If the circle passes through the point $(-2,5)$, find the equation. | **10** | **CO4** |

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|---|---|---|---|
| Q 9 | Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1}=\frac{y}{-2}=\frac{z}{3}$ and whose guiding curve is the ellipse $x^2+2y^2=1, z=3$.<br><br>**OR**<br><br>Find the equation of the right circular cylinder whose axis is $\frac{x-2}{2}=\frac{y-1}{1}=\frac{z}{3}$ and passes through the point (0, 0, 3). | **10** | **CO2** |
| **SECTION-C**<br>**(2Qx20M=40 Marks)** | | | |
| Q 10 | Define reciprocal cone and show that the cones $ax^2+by^2+cz^2=0$ and $\frac{x^2}{a}+\frac{y^2}{b}+\frac{z^2}{c}=0$ are reciprocal. | **20** | **CO2** |
| Q 11 | Show that the circles $x^2+y^2-2x-4y=0$ and $x^2+y^2-8y-4=0$ touch each other. Find the coordinates of the point of contact and the equation of the common tangents.<br><br>**OR**<br><br>Show that the general equation of the circle that passes through the point $A\,(x_{1,}\,y_1)$ and $B\,(x_{2,}\,y_2)$ may be written as $(x-x_1)(x-x_2)+(y-y_1)(y-y_2)+k\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_1 & 1 \end{vmatrix}=0.$ | **20** | **CO4** |