

Name:
Enrolment No:



UPES

End Semester Examination, December 2023

Course: B.Sc. (H) Mathematics/ Int. B. Sc. M. Sc. Mathematics
Program: BOOLEAN ALGEBRA & AUTOMATA THEORY
Course Code: MATH 3040

Semester: V
Time : 03 hrs.
Max. Marks: 100

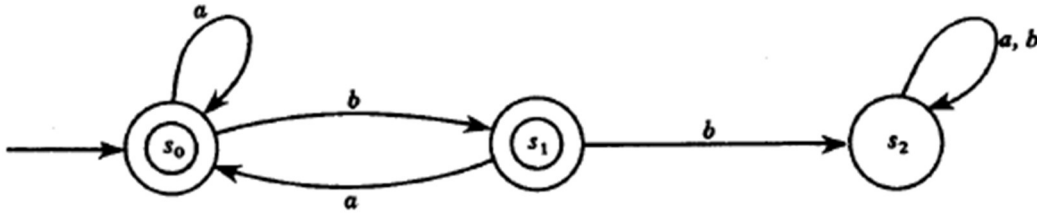
Instructions: Attempt all questions.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Let $N = \{1, 2, 3, \dots\}$ be ordered by divisibility. State whether each of the following subsets of N are linearly (totally) ordered. i) $\{24, 2, 6\}$ ii) $\{3, 15, 5\}$.	4	CO2
Q 2	Define complemented lattice with suitable example.	4	CO1
Q 3	Define regular language with suitable example.	4	CO3
Q 4	Find $\langle m \rangle$ if: i) $m = (4, 0, 3)$ ii) $m = (3, -2, 5)$.	4	CO4
Q 5	Find the prime implicants and a minimal sum-of-products form for $E = xy + xy'$.	4	CO4

SECTION B
(4Qx10M= 40 Marks)

Q 6	Find the sum of adjacent products P_1 and P_2 where: i) $P_1 = xyz'$ and $P_2 = xy'z'$. ii) $P_1 = x'yz't$ and $P_2 = x'yz't$. iii) $P_1 = xyz'$ and $P_2 = xyz't$.	10	CO2
Q 7	Consider the following languages over $A = \{a, b\}$: i) $L_1 = \{a^m b^n \mid m > 0, n > 0\}$; ii) $L_2 = \{b^m a b^n \mid m > 0, n > 0\}$. Find a regular expression r over A such that $L_i = L(r)$ for $i = 1, 2$.	10	CO3
Q 8	Determine whether the automaton M in figure given below accepts the words: $w_1 = ababba$; $w_2 = baab$.	10	CO4



Q 9 Find the prime implicants and a minimal sum-of-products form for each of the following complete sum-of-products Boolean expressions:

- (a) $E_1 = xyz + xyz' + x'y'z' + x'y'z$.
- (b) $E_2 = xyz + xyz' + xy'z + x'y'z + x'y'z$.
- (c) $E_3 = xyz + xyz' + x'y'z' + x'y'z' + x'y'z$.

OR

Show that the following are equivalent in a Boolean algebra.

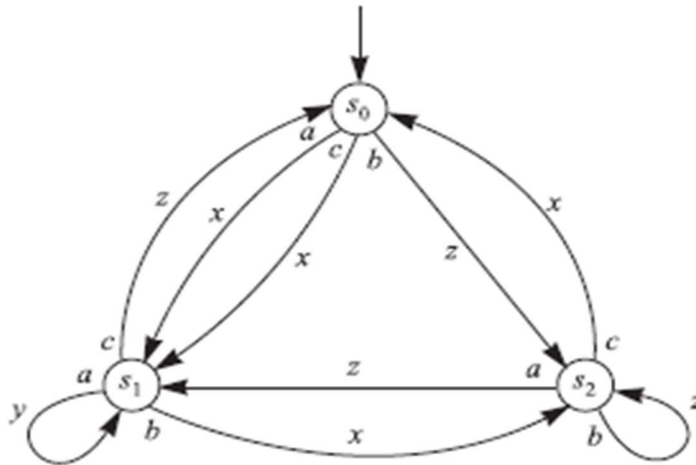
- (a) $a + b = b$ (b) $a * b = a$ (c) $a' + b = 1$ (d) $a * b' = 0$.

10

CO2

SECTION-C
(2Qx20M=40 Marks)

Q 10 Let M be the finite state machine with input set $A = \{a, b, c\}$, output set $Z = \{x, y, z\}$ and state diagram $D = D(M)$ in figure given below:



- i) Construct the state table of M .
- ii) Find the output word v if the input is the word: a) $w = a^2c^2b^2cab^3$
b) $w = ca^2b^2ac^2ab$.

20

CO5

Q 11

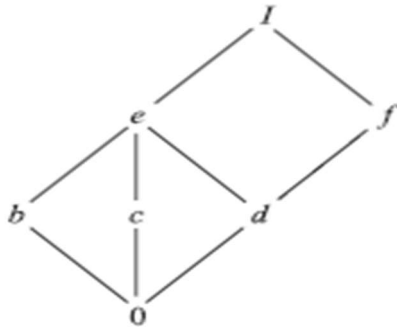
Prerequisites in college is a familiar partial ordering of available classes. We write $A < B$ if course A is a prerequisite for course B. Let C be the ordered set consisting of the mathematics courses and their prerequisites appearing in figure given below.

Class	Prerequisites
Math 101	None
Math 201	Math 101
Math 250	Math 101
Math 251	Math 250
Math 340	Math 201
Math 341	Math 340
Math 450	Math 201, Math 250
Math 500	Math 450, Math 251

- (a) Draw the Hasse diagram for the partial ordering C of these classes.
- (b) Find all minimal and maximal elements of C.
- (c) Does C have a first element or a last element?

OR

Consider the bounded lattice L in figure given below:



- (a) Find the complements, if they exist, of e and f .
- (b) Is L distributive?
- (c) Describe the isomorphisms of L with itself.

20

CO2