Name:

Enrolment No:

UPES

End Semester Examination, December 2023

Course: Complex Analysis
Semester: III
Program: B.Sc. (H) (Mathematics) | **Time: 03 hrs.**
Course Code: MATH – 2049 | **Max. Marks: 100**

Instructions: Read all the below mentioned instructions carefully and follow them strictly:
1) Mention Roll No. at the top of the question paper.
2) Attempt all the parts of a question at one place only.
3) Attempt all the questions from each section.

SECTION A
(5Qx4M=20Marks)

S. No.		**Marks**	**CO**
Q 1	Find the values of the constants a, b, c such that the following function is analytic: $f(z) = x - 2ay + i(bx - cy)$.	**4**	**CO1**
Q 3	Prove that $u(x,y) = 2x(1-y)$ is harmonic and find a function $v\,(x,y)$ such that $f(z) = u + iv$ is analytic.	**4**	**CO1**
Q 4	Using Cauchy's integral formula evaluate $\oint_C \frac{e^z}{(z+1)^4}dz$, where C is the circle $\|z\| = 3$ traversed counterclockwise.	**4**	**CO2**
Q 2	Use the Argument Principle to evaluate $\frac{1}{2\pi i}\oint_C \frac{f'(z)}{f(z)}dz$, where $f(z) = (z^2 + 1)(z-1)$ and C is the circle $\|z\| = 2$ traversed counterclockwise.	**4**	**CO2**
Q 5	Locate and classify all the singularities of $f(z) = \frac{z^8+z^4+2}{(z-1)^3(z-2)(3z+2)^2}$.	**4**	**CO3**

SECTION B
(4Qx10M= 40 Marks)

Q 6	If $f(z) = u + iv$ is an analytic function in a domain D. If any of the following conditions are satisfied, then show that $f(z)$ is constant. (a) $Arg(f(z))$ is constant. (b) $u^2 = v$.	**10**	**CO1**
Q 7	Evaluate $\int_0^{1+i}(x - y + ix^2)dz$ (a) Along the real axis from $z = 0$ to $z = 1$ and then along a line parallel to imaginary axis from $z = 1$ to $z = 1 + i$. (b) Along the imaginary axis from $z = 0$ to $z = i$ and then along a line parallel to real axis from $z = i$ to $z = 1 + i$.	**10**	**CO2**

Q 8	Using the residue theorem evaluate $\frac{1}{2\pi i}\oint_C \frac{e^{2z}}{z^2(z^2+2z+2)}dz$ where C is the circle $\|z\| = 3$ traversed counterclockwise.	**10**	**CO4**
Q 9	Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in the Laurent series valid for (a) $0 < \|z+1\| < 2$ (b) $\|z\| < 1$. **OR** Discuss the singularities of the function $f(z) = \frac{e^z}{z^2(1-\cos z)}$ and classify them.	**10**	**CO3**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	(a) If $f(z)$ has a pole of order k at $z = a$ then prove that residue (a_{-1}) is given by $a_{-1} = \frac{1}{(k-1)!}\lim_{z\to a}\frac{d^{k-1}}{dz^{k-1}}(z-a)^k f(z)$. (b) Find the residue of $f(z) = \frac{1}{(z-2)(z-3)(z-4)}$ at all its poles in the complex plane $\mathbb{C}$ and evaluate $\oint_C f(z)\,dz$, where C is $\|z-i\| = \pi$ traversed counterclockwise.	**20**	**CO4**
Q 11	(a) If $f(z)$ is analytic inside a circle $\mathbb{C}$ with center a, then for all z inside $\mathbb{C}$ then show that $f(z) = f(a) + (z-a)f'(a) + \frac{(z-a)^2}{2!}f''(a) + + \frac{(z-a)^3}{3!}f'''(a) + \cdots$ (b) Expand $f(z) = \cos z$ in Taylor series up to three terms about $z = \frac{\pi}{4}$. **OR** Let $f(z) = \ln(1+z)$ then (a) Expand $f(z)$ in a Taylor series about $z = 0$. (b) Determine the region of convergence for the series in (a). (c) Expand $\ln\left(\frac{1+z}{1-z}\right)$ in a Taylor series about $z = 0$.	**20**	**CO3**