


Name:			
Enrolment No:			
UPES End Semester Examination, December 2023			
Course: Engineering Mathematics I Program: B. Tech. [ASE+APE(UP)+ADE+Chemical+E&CE+Civil+ Mechatronics+ Mechanical +Electronics & Communication] Course Code: MATH 1050		Semester: I Time : 03 hrs. Max. Marks: 100	
Instructions: All questions are compulsory.			
SECTION A (5Qx4M=20Marks)			
S. No.		Marks	CO
Q 1	Find the rank of matrix $A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 2 \end{bmatrix}$	4	CO1
Q 2	Evaluate $\int_0^{\infty} x^4 e^{-\sqrt{x}} dx$.	4	CO2
Q 3	If $u = x^2 + y^2 + z^2$, prove that $xu_x + yu_y + zu_z = 2u$.	4	CO2
Q 4	Find $\text{curl}(\text{curl}\vec{V})$ where $\vec{V} = 2xz^2\hat{i} - yz\hat{j} + 3xz^3\hat{k}$ at $(1, 1, 1)$.	4	CO3
Q 5	Evaluate $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = x^2\hat{i} + xy\hat{j}$ and C is the boundary of the square in the plane $z = 0$ and bounded by $x = 0, y = 0, x = 1$ and $y = a$.	4	CO3
SECTION B (4Qx10M= 40 Marks)			
Q 6	Let $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$. Find the modal matrix P such that $P^{-1}AP$ is a diagonal matrix.	10	CO1
Q 7	Evaluate $\iint_R (x + y) dy dx$, where R is the region bounded by the lines $x = 0, x = 2, y = x$ & $y = x + 2$.	10	CO2
Q 8	If the vector $\vec{F} = (ax^2y + yz)\hat{i} + (xy^2 - xz^2)\hat{j} + (2xyz - 2x^2y^2)\hat{k}$ is solenoidal, find the value of a . Also find the curl of this solenoidal vector.	10	CO3

Q 9	Find the Fourier series representing $f(x) = x$, $0 < x < 2\pi$. OR Using Maclaurin's series, expand $\log(1+x)$. Hence, deduce that $\log \sqrt{\frac{1+x}{1-x}} = x + \frac{x^3}{3} + \frac{x^5}{3} + \dots$	10	CO4
SECTION-C (2Qx20M=40 Marks)			
Q 10A	If $u = x + y + z, v = x^2 + y^2 + z^2, w = yz + zx + xy$, prove that $\text{grad } u, \text{grad } v$ and $\text{grad } w$ are coplanar vectors. OR Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$.	10	CO3
Q 10B	If a force $\vec{F} = 2x^2y\hat{i} + 3xy\hat{j}$ displace a particle in the xy plane from $(0, 0)$ to $(1, 4)$ along a curve $y = 4x^2$, find the work done. OR Apply the Green's theorem to evaluate $\oint_C (2x^2 - y^2) dx + (x^2 + y^2) dy$, where C is the boundary of the region enclosed by x -axis and the upper half of the circle $x^2 + y^2 = a^2$	10	CO3
Q 11	Find the Fourier series for $f(x)$, if $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$. Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$.	20	CO4