Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UPES

End Semester Examination, May 2023

Course: Probability and Statistics **Semester: IV**

Program: B.Sc (H) Physics / Chemistry / Geology **Time: 03 hrs.**

Course Code: MATH2020G **Max. Marks: 100**

Instructions: Attempt All Questions.

SECTION A

(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Let $\Omega = \{1,2,3,4\}$ be a sample space. Check whether the set $\mathcal{F} = \{\phi, \Omega, \{1\}, \{2,3,4\}, \{3,4\}\}$ is a sigma field. If not, then write down the smallest sigma field containing $\mathcal{F}$.	**4**	**CO1**
Q 2	The coefficients of the equation $ax^2 + bx + c = 0$ are determined by throwing an ordinary die. What is the probability that the framed equation will have real roots?	**4**	**CO1**
Q 3	Suppose that a random variable X has moment generating function $M_X(t) = \left(\frac{1}{2}\right)e^{-5t} + \left(\frac{1}{6}\right)e^{4t} + \left(\frac{1}{8}\right)e^{5t} + \left(\frac{5}{24}\right)e^{25t}$. Evaluate the standard deviation of X.	**4**	**CO2**
Q 4	Suppose that 50 people live in a building. The parking lot has the capacity for 30 cars. If each person has a car with probability 1/2 (no one has more than one car), what is the probability that there won't be enough parking spaces for all the cars?	**4**	**CO3**
Q 5	The marks obtained by students in a batch are distributed with mean 120 and standard deviation 5 (variance 25). B grades are awarded to students with marks between 112 and 128. Using Chebyshev's inequality, find a lower bound on the probability of a student getting B grade.	**4**	**CO4**

SECTION B

(4Qx10M= 40 Marks)

S. No.		Marks	CO
Q 6	Suppose that a random variable X has a distribution function $$F(x) = \begin{cases} a + be^{x}, & if\ x < 0 \\ \dfrac{x^2}{4\pi^2}, & if\ \ 0 \le x < 2\pi \\ c + de^{-x}, & if\ x \ge 2\pi \end{cases}$$	**10**	**CO1**

	Where $a, b, c,$ and d are constants. Find the values of a, b, c and d. Derive the probability density/mass function of X. Also derive probability density / mass function of $Y = \cos(X)$.		
Q 7	The length of similar components produced by a company are approximated by a normal distribution model with a mean of 5 cm and a standard deviation of 0.02 cm. If a component is chosen at random then compute the probability that the length of the component is a) between 4.98 and 5.02 cm. b) between 4.96 and 5.04 cm.	**10**	**CO1**
Q 8	A company makes a certain device, and we are interested in its lifetime. The lifetime is exponentially distributed with a parameter $\lambda = 2$ years. Let X be the lifetime of a randomly chosen device. Find the generalized pdf of X and evaluate $P(X \geq 1)$, $P(X > 2 \mid X \geq 1)$, $E(X)$, and $Var(X)$.	**10**	**CO2**
Q 9	For each of the following random variables, Evaluate $P(X > 5)$, and $P(2 < X \leq 6)$. $(i)\ X \sim Geometric\left(\frac{1}{5}\right), \left(f_X(x) = p(1-p)^{k-1} I_{\{1,2,\ldots\}}\right)$ $(ii)\ X \sim Binomial\left(\frac{10,1}{3}\right)$, $(iii)\ X \sim Hypergeometric\ (10,10,12), \left(f_X(x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}} I_{\{\max\{0,n+M-N\},\ldots,\min\{n,M\}\}}\right)$ $(iv)\ X \sim Poisson\ (5)$. **OR** In each of 25 state elections, party A has a 20% chance of winning. Use the binomial and normal approximation to binomial to calculate the probability that party A will (a) win exactly 23 states. (b) win 4 or fewer states. (c) win exactly 10 states. (d) win 10 or fewer states.	**10**	**CO2**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	Suppose 50% of the population approves of the job the governor is doing, and that 20 individuals are drawn at random from the population. Using both the binomial distribution and the normal approximation to the binomial calculate the probability that (a) exactly 7 people will support the governor. (b) 7 or fewer people will support the governor. (c) exactly 11 will support the governor. (d) 11 or fewer will support the governor.	**20**	**CO3**
Q 11	Random variables X and Y have joint continuous distribution with p.d.f. $f(x,y) = c(x+3y)e^{-x-2y}, if\ x,y \geq 0\ and = 0\ therwise$. (a) Find the value of c. (b) Compute $E[X]$ and $E[Y]$. (c) Formulate $E[X \mid Y = y]$. **OR** Random variables X and Y have joint continuous distribution with p.d.f. $f(x,y) = cx + 1, if\ x,y \geq 0,\ x + y < 1;\ and = 0\ therwise$. (a) Find the value of c. (b) Compute $E[X]$ and $E[Y]$ and $P(Y < 2X^2)$.	**20**	**CO4**