


Name:			
Enrolment No:			
UNIVERSITY OF PETROLEUM AND ENERGY STUDIES End Semester Examination, May 2023			
Course: Engineering Mathematics II Program: B. Tech. (APE-US, ADE, CHE, AE, APE-GS, ME, MECHATONICS, ECE, ELECTRO-CSE) Course Code: MATH1051 Instructions: All questions are compulsory.		Semester: II Time : 03 hrs. Max. Marks: 100	
SECTION A (5Qx4M=20Marks)			
S. No.		Marks	CO
Q1	Solve the following differential equation $(D^2 - 3D + 2)y = e^{5x}$, where $D \equiv \frac{d}{dx}$	4	CO1
Q2	If $w = \ln z$ ($z = x + iy$), find $\frac{dw}{dz}$ and determine where w is non-analytic.	4	CO2
Q3	Prove that $\int_C \frac{dz}{z-a} = 2\pi i$, where C is the circle $ z - a = r$	4	CO2
Q4	Find the nature and location of singularities of the following function $\frac{z - \sin z}{z^2}$	4	CO3
Q5	Eliminate arbitrary constants a and b from $z = (x - a)^2 + (y - b)^2$ to form the partial differential equation.	4	CO4
SECTION B (4Qx10M= 40 Marks)			
Q6	Test whether the equation $(x + y)^2 dx - (y^2 - 2xy - x^2) dy = 0$ is exact and hence solve it.	10	CO1
Q7	Evaluate, using Cauchy's integral formula: $\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$, where C is the circle $ z = 3$	10	CO2
Q8	Expand the function $f(z) = \sin z$ in a Taylor's series about $z = 0$ and determine the region of convergence.	10	CO3
Q9	Solve the following partial differential equation $\left(\frac{y^2 z}{x}\right) \frac{\partial z}{\partial x} + (xz) \frac{\partial z}{\partial y} = y^2$	10	CO4
OR			

	By using Lagrange's method find the solution of the partial differential equation $y^2 \frac{\partial z}{\partial x} - xy \frac{\partial z}{\partial y} = x(z - 2y)$		
SECTION-C (2Qx20M=40 Marks)			
Q10A	By integrating around a unit circle, evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{1-2a \cos \theta+a^2} d\theta$, where $a^2 < 1$.	10	CO3
Q10B	Find Taylor's series expansion of $f(z) = \frac{1}{(z+1)^2}$ about the point $z = -i$.	10	CO3
Q11	Determine the solution of one-dimensional heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$, $0 < x < L$, under the following conditions boundary conditions: $u(0, t) = u(L, t) = 0$ for all $t > 0$. Initial condition: $u(x, 0) = f(x)$. <p style="text-align: center;">OR</p> Using method of separation of variables solve the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$, $0 < x < L$, subject to the boundary conditions: $u(0, t) = u(L, t) = 0$ for all $t > 0$ and initial conditions: $u(x, 0) = f(x)$ and $u_t(x, 0) = g(x)$.	20	CO4