Name:

Enrolment No:

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, May 2022

Course: Linear Programming and Theory of Games **Semester: VI**
Program: BSc(H) Mathematics **Time : 03 hrs.**
Course Code: MATH 3016 **Max. Marks: 100**

Instructions: All questions are compulsory

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Find the range of values of p and q that will render the entry $(2,2)$ a saddle point for the game Player B Player A | B_1 | B_2 | B_3 A_1 | 2 | 4 | 5 A_2 | 10 | 7 | q A_3 | 4 | p | 6	**4M**	**CO5**
Q 2	Write the dual to the following LP problem. Maximize $Z = x_1 - x_2 + 3x_3$ subject to the constraints $x_1 + x_2 + x_3 \leq 10,$ $2x_1 - x_2 - x_3 \leq 2,$ $2x_1 - 2x_2 - 3x_3 \leq 6,$ And $x_1, x_2, x_3 \geq 0$	**4M**	**CO2**
Q 3	A manufacturer produces two types of models M_1 and M_2, each model of type M_1 requires 4 hours for grinding and 2 hours for polishing. Each model of M_2 requires 2 hours of grinding and 5 hours of polishing. The manufacturer has 2 grinders and 3 polishers, each grinder work 40 hours a week and polisher 60 hour a week. Profit on M_1 is Rs3 and on model M_2 is Rs4. Formulate Linear Programming Problem.	**4M**	**CO1**

<table>
<tr><td>Q 4</td><td>Determine an initial basic feasible solution to the following transportation problem using North West Corner Rule

Destination
Source
<table><tr><th></th><th>D_1</th><th>D_2</th><th>D_3</th><th>D_4</th><th>Supply</th></tr><tr><td>S_1</td><td>21</td><td>16</td><td>15</td><td>3</td><td>11</td></tr><tr><td>S_2</td><td>17</td><td>18</td><td>14</td><td>23</td><td>13</td></tr><tr><td>S_3</td><td>32</td><td>27</td><td>18</td><td>41</td><td>19</td></tr><tr><td>Demand</td><td>6</td><td>6</td><td>8</td><td>23</td><td></td></tr></table></td><td>4M</td><td>CO4</td></tr>
<tr><td>Q 5</td><td>For what value of λ, the game with following pay-off matrix is strictly determinable.

Player B
<table><tr><td>Player A</td><td>B_1</td><td>B_2</td><td>B_3</td></tr><tr><td>A_1</td><td>λ</td><td>6</td><td>2</td></tr><tr><td>A_2</td><td>-1</td><td>λ</td><td>-7</td></tr><tr><td>A_3</td><td>-2</td><td>4</td><td>λ</td></tr></table></td><td>4M</td><td>CO5</td></tr>
<tr><td colspan="4">SECTION B
(4Qx10M= 40 Marks)</td></tr>
<tr><td>Q 6</td><td>Use the Simplex method to solve the following LP Problem
$Maximize\ Z = 3x_1 + 2x_2$
Subject to the constraints
$x_1 + x_2 \leq 4$
$x_1 - x_2 \leq 2$
And $x_1, x_2 \geq 0$.</td><td>10M</td><td>CO2</td></tr>
<tr><td>Q 7</td><td>A department of a company has five employees with five jobs to be performed. The time (in hours) that each man takes to perform each job is given in the effectiveness matrix.

<table><tr><td></td><td colspan="6">Employees</td></tr><tr><td rowspan="6">Jobs</td><td></td><td>I</td><td>II</td><td>III</td><td>IV</td><td>V</td></tr><tr><td>A</td><td>10</td><td>5</td><td>13</td><td>15</td><td>16</td></tr><tr><td>B</td><td>3</td><td>9</td><td>18</td><td>13</td><td>6</td></tr><tr><td>C</td><td>10</td><td>7</td><td>2</td><td>2</td><td>2</td></tr><tr><td>D</td><td>7</td><td>11</td><td>9</td><td>7</td><td>12</td></tr><tr><td>E</td><td>7</td><td>9</td><td>10</td><td>4</td><td>12</td></tr></table></td><td>10M</td><td>CO3</td></tr>
</table>

<table>
<tr><td></td><td>How the jobs should be allocated, one per employee as to minimize the total man-hours.</td><td></td><td></td></tr>
<tr><td>Q 8</td><td>ABC Ice Cream Company has a distribution depot in Greater Kailash Part I for distributing ice-cream in South Delhi. There are four vendors located in different parts of South Delhi (call them A, B, C and D) who have to be supplied ice-cream every day. The following matrix displays the distances (in kilometers) between the depot and the four vendors.

<table>
<tr><td colspan="7">To</td></tr>
<tr><td rowspan="6">From</td><td></td><td>Depot</td><td>Vendor A</td><td>Vendor B</td><td>Vendor C</td><td>Vendor D</td></tr>
<tr><td>Depot</td><td>∞</td><td>3.3</td><td>3</td><td>4</td><td>2</td></tr>
<tr><td>Vendor A</td><td>3.5</td><td>∞</td><td>4</td><td>2.5</td><td>3</td></tr>
<tr><td>Vendor B</td><td>3</td><td>4</td><td>∞</td><td>4.5</td><td>3.5</td></tr>
<tr><td>Vendor C</td><td>4</td><td>2.5</td><td>4.5</td><td>∞</td><td>4</td></tr>
<tr><td>Vendor D</td><td>2</td><td>3</td><td>3.5</td><td>4</td><td>∞</td></tr>
</table>
What route should the company van follow so that the total distance travelled is minimized?</td><td>10M</td><td>CO4</td></tr>
<tr><td>Q 9</td><td>A company management and the labor union are negotiating a new three year settlement. Each of these has 4 strategies
i. Hard and aggressive bargaining
ii. Reasoning and logical approach
iii. Legalistic strategy
iv. Conciliatory approach.
The costs to the company are given for every pair of strategy choice.

Company Strategies
<table>
<tr><td>Union Strategies↓</td><td>I</td><td>II</td><td>III</td><td>IV</td></tr>
<tr><td>I</td><td>20</td><td>15</td><td>12</td><td>35</td></tr>
<tr><td>II</td><td>25</td><td>14</td><td>8</td><td>10</td></tr>
<tr><td>III</td><td>40</td><td>2</td><td>10</td><td>5</td></tr>
<tr><td>IV</td><td>-5</td><td>4</td><td>11</td><td>0</td></tr>
</table>
Which strategy will the two side adopt? Also determine the value of game

OR</td><td>10M</td><td>CO5</td></tr>
</table>

Solve the game whose payoff matrix is given below

Player B

Player A	B_1	B_2	B_3	B_4
A_1	3	2	4	0
A_2	3	4	2	4
A_3	4	2	4	0
A_4	0	4	0	8

SECTION-C
(2Qx20M=40 Marks)

Q 10

A company has three production factoriesF_1, F_2 and F_3 with production capacity of 7,9 and 18 units per week of product respectively. These units are to be shipped to four warehousesW_1,W_2, W_3 and W_4 with requirement of 5,8,7 and 14 units per week, respectively. The transportation costs per unit between factories to warehouses are given in the table below. Find the initial basic feasible solution of the following transportation problem by Vogel's Approximation method

Ware house→ Factory↓	W_1	W_2	W_3	W_4	Factory Capacity
F_1	19	30	50	10	7
F_2	70	30	40	60	9
F_3	40	8	70	20	18
Warehouse Requirement	5	8	7	14	34

Is the above solution an optimal solution? If not, obtain the optimal solution.

20M **CO4**

Q 11	Use two-phase simplex method to solve the following LP problem *Minimize* $Z = x_1 + x_2$ Subject to the constraints $2x_1 + x_2 \geq 4$ $x_1 + 7x_2 \geq 7$ And $x_1, x_2 \geq 0$. **OR** Use Big M method to solve the following LP problem *Minimize* $Z = 5x_1 + 3x_2$ Subject to the constraints $2x_1 + 4x_2 \leq 12$ $2x_1 + 2x_2 = 10$ $5x_1 + 2x_2 \geq 10$ And $x_1, x_2 \geq 0$.	**20M**	**CO2**