Name: **Enrolment No:**	

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, May 2022

Course: Real Analysis **Semester: II**
Program: B.Sc. (H) Mathematics **Time : 03 hrs.**
Course Code: MATH 1018 **Max. Marks: 100**

Instructions: Attempt all questions. All questions are compulsory.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Determine the supremum and infimum, if they exist, of the following sets: **(i).** $\left\{sin\frac{n\pi}{6} : n \in N\right\}$ **(ii).** $\{x \in R : 2x + 5 > 0\}$	**4**	**CO1**
Q 2	Determine the limit points of the following sets: (i). $S = \left\{\frac{1}{n} : n \in N \right\}$. (ii). $S = \left\{\left(1 - \frac{1}{n}\right) sin\frac{n\pi}{2} : n \in N \right\}$.	**4**	**CO1**
Q 3	If $S_n = \frac{(n+1)(n^3-n)}{(n^2+2)(n^2+1)}$, then determine the value of $\lim_{n\to\infty} S_n$.	**4**	**CO2**
Q 4	Determine the value of $\lim_{n\to\infty} \left(\frac{(3n)!}{(n!)^3}\right)^{\frac{1}{n}}$.	**4**	**CO2**
Q 5	Apply Cauchy's integral test to examine the convergence of $\sum_{n=1}^{\infty} \frac{1}{n+n^2}$.	**4**	**CO3**

SECTION B
(4Qx10M= 40 Marks)

Q 6	Show that the interior of a set ***S*** is the largest open subsets of ***S***.	**10**	**CO1**
Q 7	Show that the set of all real numbers in the closed interval [0, 1] is not a countable set.	**10**	**CO1**
Q 8	State and prove Cauchy's first theorem on limits.	**10**	**CO2**

Q 9	Show that the series $\sum_{n=1}^{\infty}(2)^n \sin\frac{x}{3^n}$ is absolutely convergent for all finite values of x. **OR** Examine the convergence of the alternating series $\frac{1}{1.2}-\frac{1}{3.4}+\frac{1}{5.6}-\frac{1}{7.8}+\cdots$	**10**	**CO3**
SECTION-C **(2Qx20M=40 Marks)**			
Q 10	**(i).** Show, by applying Cauchy's convergence criterion, that the sequence $< S_n >$ defined by $S_n = 1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}$ does not converge. **(ii).** Examine the convergence of the sequence $< S_n >$ defined by $S_n = \frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\cdots+\frac{1}{n!}$ by applying Cauchy's convergence criterion.	**20**	**CO2**
Q 11	Discuss the convergence of the following series **(i).** $\frac{1^2}{2^2}+\frac{1^2.3^2}{2^2.4^2}x+\frac{1^2.3^2.5^2}{2^2.4^2.6^2}x^2+\cdots$ **(ii).** $\sum_1^{\infty}\left[\sqrt{(n^2+1)}-n\right] x^{2n}$ **OR** Discuss the convergence of the following series **(i).** $1+\frac{3\,x}{7}+\frac{3.\ 6}{7.\ 10}x^2+\frac{3.\ 6.\ 9}{7.\ 10.\ 13}x^3+\frac{3.\ 6.\ 9.\ 12}{7.\ 10.\ 13.\ 16}x^4+\cdots$ **(ii).** $x\log x + x^2 \log 2x + x^3 \log 3x + \cdots + x^n \log nx + \ldots$	**20**	**CO3**