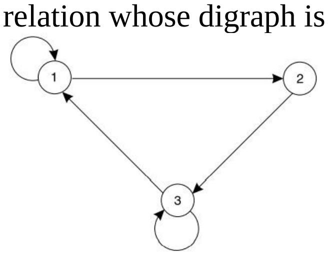
Name: **Enrolment No:**	

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2022

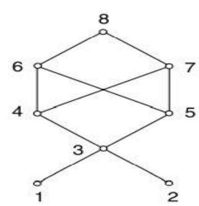
Course: Mathematics III **Semester: III**
Program: B. Tech (Civil Engineering) **Time : 03 hrs.**
Course Code: MATH2045 **Max. Marks: 100**
Instructions: All questions are compulsory.

SECTION A
(5Qx4M=20Marks)

S. No.		Marks	CO
Q 1	Find the Laplace transform of $$f(t)=\begin{cases} t, & 0<t<\frac{1}{2} \\ t-1, & \frac{1}{2}<t<1 \\ 0, & t>1 \end{cases}$$	**4**	**CO1**
Q 2	If $F(s)=\frac{1}{s^2(s+5)}$ find the Inverse Laplace transform.	**4**	**CO1**
Q3	Determine a truth table of $\neg p \rightarrow (q \rightarrow p)$	**4**	**CO2**
Q4	Determine whether the relation whose digraph is given below is a poset 	**4**	**CO3**
Q5	Solve the recurrence relation $a_r - 7a_{r-1} + 10a_{r-2} = 0$ for $r \geq 2$ by method of generating functions.	**4**	**CO4**

SECTION B
(4Qx10M= 40 Marks)

Q6	Prove the following logical equivalencies: **(a)** $p \vee [p \wedge (p \vee q)] \equiv p$ **(b)** $[(\neg p \vee q) \wedge (p \wedge (p \wedge q))] \equiv p \wedge q$	**10**	**CO2**
Q7	Draw the Hasse diagram representing the positive divisions of *36* **OR** If $A=\{3,4,12,24,48,72\}$ and the relation $\leq$ be such that $a \leq b$ if a divides b. Draw the Hasse diagram of $(A,\leq)$.	**10**	**CO3**

Q8	Show that if x and y are elements of a lattice L then $x \vee y = y$ if and only if $x \wedge y = x$.	**10**	**CO3**
Q9	Consider a second-order homogeneous recurrence relation $a_n = a_{n-1} + 2a_{n-2}$ with initial conditions $a_0 = 2$, $a_1 = 7$, **(a)** Find the next three term of the sequence. **(b)** Find the general solution. **(c)** Find the unique solution with the given initial conditions.	**10**	**CO4**
SECTION-C **(2Qx20M=40 Marks)**			
Q10A	Evaluate $L\{f(t)\}$ and $L\{f'(t)\}$ of $$f(t) = \begin{cases} t, & 0 \leq t < 3 \\ 6, & t > 3 \end{cases}$$ **OR** Find the z-transform of the $f(k) = \sin \alpha k,\ k \geq 0$.	**10**	**CO1**
Q10B	Solve the differential equation $$\frac{d^2y}{dt^2} + 9y = \cos 2t, y(0) = 1, y\left(\frac{\pi}{2}\right) = -1$$ **OR** Find the inverse $\boldsymbol{z}$-transform of $\frac{\boldsymbol{z}}{\boldsymbol{z-a}}$ when **(a)** $\lvert z \rvert > \lvert a \rvert$ **(b)** $\lvert z \rvert < \lvert a \rvert$.	**10**	**CO1**
Q11A	Prove that if $(A, \leq)$ and $(B, \leq)$ are partially order sets, then $(A \times B, \leq)$ is a partially ordered set with partial order $\leq$, defined by $(a, b) \leq (a', b')$ if $a \leq a'$ in A and $b \leq b'$ in B.	**10**	**CO3**
Q11B	Consider the poset $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ whose Hasse diagram is shown in the following Figure  and let $B = \{3, 4, 5, 6\}$. Find **(a)** Upper bounds of B, **(b)** Lower bounds of B, **(c)** Greatest lower bound of B, **(d)** Least upper bound of B.	**10**	**CO3**