Name:

Enrolment No:

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2022

Program Name : B. Sc. (H) Mathematics & Integrated B.Sc. Mathematics
Course Name : Ordinary Differential Equations **Semester : III**
Course Code : MATH 2048 **Time : 03 Hrs.**
Nos. of page(s) : 02 **Max Marks : 100**
Instructions:
Attempt all questions from Section A (each carrying 4 marks); attempt all questions from Section B (Each carrying 10 marks) and attempt all questions from Section C (each carrying 20 marks). Question 8 and 11 have internal choice.

SECTION A

S. No.		**Marks**	**CO**
Q 1	Under what condition the following differential equation $$(ax+y)dx+(kx+by)dy=0$$ is exact.	**4**	**CO1**
Q 2	Find the general solution and singular solution(s) of the differential equation $8ap^3=27y$.	**4**	**CO2**
Q 3	Find the particular integral of the differential equation $$(D^4+3D^2)y=108x^2;\ D \text{ stands for } \frac{d}{dx}.$$	**4**	**CO3**
Q 4	When a switch is closed in circuit containing a battery E, a resistor R and an inductance L, the current i builds up at a rate given by $$L\frac{di}{dt}+Ri=E.$$ Find i as a function of t.	**4**	**CO4**
Q 5	Classify the critical point $(0,0)$ of the linear system $X'=AX$ where $$A=\begin{bmatrix}-10 & 6\\ 15 & -19\end{bmatrix}.$$	**4**	**CO5**

SECTION B

Q 6	Show that the equation of the curve whose differential equation is $$p^2+2py\cot x=y^2$$ and passing through the point $\left(\frac{\pi}{2},1\right)$ is $\left[2y-\sec^2\left(\frac{x}{2}\right)\right]\left[2y-\csc^2\left(\frac{x}{2}\right)\right]$.	**10**	**CO2**

Q 7	Reduce the differential equation $(px^2 + y^2)(px + y) = (p + 1)^2$ to Clairaut's form by the substitutions $u = xy$, $v = x + y$ and then obtain the complete primitive.	**10**	**CO2**
Q 8	Apply the method of variation of parameters to solve the differential equation $$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = x + \cos x.$$ **OR** Using the method of undetermined coefficients to solve the following differential equation: $$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 2e^x - 10\sin x.$$	**10**	**CO3**
Q 9	A body whose temperature T is initially $200^0 C$ is immersed in a liquid when temperature T is constantly $100^0 C$. If the temperature of the body is $150^0 C$ at $t = 1$ minute, what is the temperature at $t\ = 2$ minutes?	**10**	**CO4**
SECTION-C			
Q 10	If $M(x,y)dx + N(x,y)dy = 0$ and $P(x,y)dx + Q(x,y)dy = 0$ are exact differential equations, then show that $(M + P)dx + (N + Q)dy = 0$ is also an exact differential equation. Also, Solve the differential equation $$(3x^2y^3e^y + y^3 + y^2)dx + (x^3y^3e^y - xy)dy = 0.$$	**20**	**CO1**
Q 11	Solve the Cauchy-Euler homogeneous differential equation $$x^2\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + y = x^{-1}[1 + \log_e x \sin(\log_e x)], x > 0.$$ **OR** Define Wronskian. Show that the Wronskian of the functions x^2 and $x^2 \log_e x$ is non zero. Can these functions be independent solutions of an ordinary differential equation? If so, determine the differential equation.	**20**	**CO3**