Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2022

Course: Finite Element Method for Fluid Dynamics **Semester: I**
Program: M. Tech Computational Fluid Dynamics (CFD) **Time: 03 hrs.**
Course Code: ASEG 7022 **Max. Marks: 100**

Instructions: Make use of sketch/plots to elaborate your answer. All sections are compulsory.

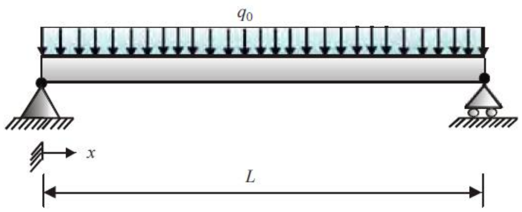
SECTION A
(5Qx4M=20Marks)

S. No.	**Provide a brief explanation for your choice.**	**Marks**	**CO**
Q 1	Which relations are used in one dimensional finite element modeling? a) Stress-strain relation b) Strain-displacement relation c) Total potential energy d) Total potential energy; Stress-strain relation; Strain-displacement relation.	**[04]**	**CO2**
Q 2	Stiffness matrix represents a system of ________ a) Programming equations b) Iterative equations c) Linear equations d) Program CG SOLVING equations	**[04]**	**CO1**
Q 3	What are the basic unknowns on stiffness matrix method? a) Nodal displacements b) Vector displacements c) Load displacements d) Stress displacements	**[04]**	**CO1**
Q 4	Write the element stiffness matrix for a beam element. a) $K = \frac{2EI}{l}$ b) $K = \frac{2EI}{l}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ c) $K = \frac{2E}{l}\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ d) $K = \frac{2E}{l}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$	**[04]**	**CO2**

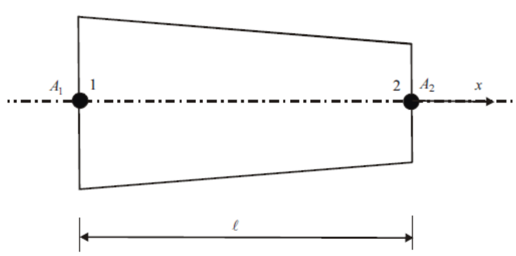
<table>
<tr><td>Q 5</td><td>Principal of minimum potential energy follows directly from the principal of ________
a) Elastic energy
b) Virtual work energy
c) Kinetic energy
d) Potential energy</td><td>[04]</td><td>CO3</td></tr>
<tr><td colspan="4">SECTION B
(4Qx10M= 40 Marks)</td></tr>
<tr><td>Q 6</td><td>Consider a simply supported beam under uniformly distributed load as shown in figure. The governing differential equation and the boundary conditions are given by

$$EI\frac{d^4v}{dx^4} - q_0 = 0$$

$$v(0) = 0, \frac{d^2v}{dx^2}(0) = 0, v(L) = 0, \frac{d^2v}{dx^2}(L) = 0$$



Find the approximate solution using the point collocation technique at $x = L/2$.</td><td>[10]</td><td>CO2</td></tr>
<tr><td>Q 7</td><td>Consider a bar element whose area of cross-section varies linearly along the longitudinal axis. Derive its stiffness matrix. How will this compare with the stiffness matrix obtained assuming that the bar is of uniform cross section area equal to that of its mid-length?


</td><td>[10]</td><td>CO3</td></tr>
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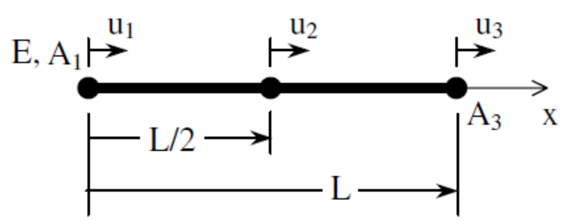
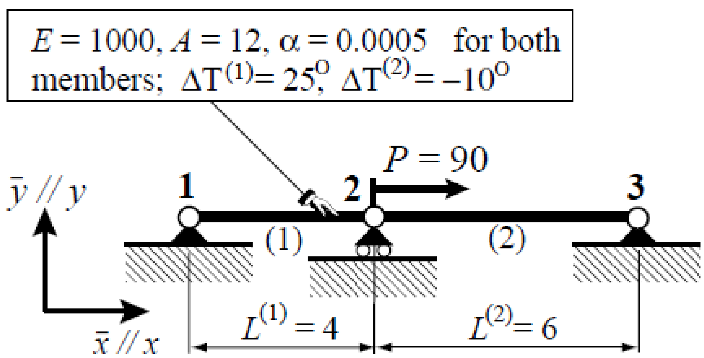
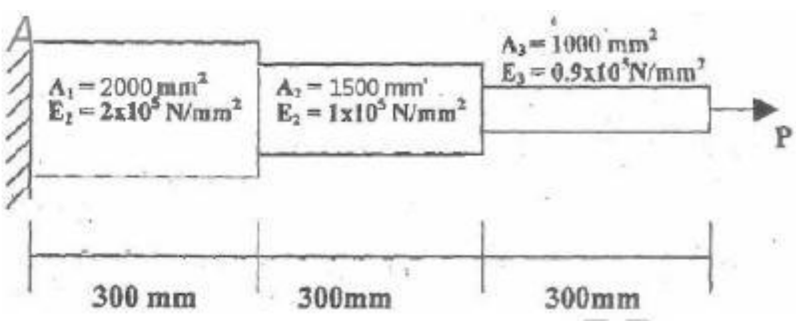
Q 8	A 3 node rod element has a quadratic shape function matrix: $$N = \langle 1 - \frac{3x}{L} + \frac{2x^2}{L^2}, \frac{4x}{L} - \frac{4x^2}{L^2}, -\frac{x}{L} + \frac{2x^2}{L^2} \rangle$$ For L = 1 m, E = 200 × 10^9 Pa, $u_1 = 0, u_2 = 5 \times 10^{-6} m, u_2 = 15 \times 10^{-6} m$ Find: a. The displacement u at $x = 0.25\ m$. b. The strain as a function of x. c. The strain at $x = 0.25\ m$. d. The stress at $x = 0.25\ m$ 	**[10]**	**CO3**
Q 9	Considering the following force displacement relationship, $$\mathbf{f} = \mathbf{Ku} = \begin{bmatrix} 20 & 10 & -10 & 0 & -10 & -10 \\ 10 & 10 & 0 & 0 & -10 & -10 \\ -10 & 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & -5 \\ -10 & -10 & 0 & 0 & 10 & 10 \\ -10 & -10 & 0 & -5 & 10 & 15 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0.4 \\ -0.2 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \\ 0 \\ 1 \\ 2 \\ 1 \end{bmatrix}$$ Draw a free body diagram of the nodal forces acting on the free-free truss structure and verify that this force system satisfies translational and rotational (moment) equilibrium. **OR** Solve the differential equation for a physical problem expressed as $\frac{d^2y}{dx^2} + 100 = 0$ 0 ≤ x ≤ 10 with boundary conditions as y(0)=0 and y(10)=0 using (i) Point collocation method (ii) Sub domain collocation method	**[10]**	**CO4**

<table>
<tr><td colspan="4">SECTION-C
(2Qx20M=40 Marks)</td></tr>
<tr><td>Q 10</td><td>Two truss members are connected in series as shown in fig and fixed at the ends. Properties $E = 1000$, $A = 12$ and $\alpha = 0.0005$ are common to both members. The member lengths are 4 and 6. A mechanical load $P = 90$ acts on the roller node. The temperature of member (1) increases by $\Delta T(1) = 25^0$ while that of member (2) drops by $\Delta T(2) = -10^0$. Find the stress in both members.
</td><td>[20]</td><td>CO5</td></tr>
<tr><td>Q 11</td><td>Consider the bar shown in figure axial force P = 30KN is applied as shown. Determine the nodal displacement, stresses in each element and reaction forces.

OR
Derive the equivalent spring formula $F = (EA/L)\ d$ by the Theory of Elasticity relations $e = d\bar{u}(\bar{x})/d\bar{x}$ (strain-displacement equation), $\sigma = Ee$ (Hooke's law) and $F = A\sigma$ (axial force definition). Here e is the axial</td><td>[20]</td><td>CO5</td></tr>
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	strain (independent of *x*) and σ the axial stress (also independent of *x*). Finally, *u(x)* denotes the axial displacement of the cross section at a distance *x* from node *i* , which is linearly interpolated as $$\bar{u}(\bar{x}) = \bar{u}_{xi}\left(1 - \frac{\bar{x}}{L}\right) + \bar{u}_{xj}\frac{\bar{x}}{L}$$ Justify above equation is correct since the bar differential equilibrium equation: $d[A(d\sigma/d\bar{x})]/d\bar{x} = 0$, is verified for all *x* if *A* is constant along the bar.		