Name: **Enrolment No:**	UPES UNIVERSITY OF TOMORROW

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2022

Course: Mathematical Physics **Semester: I**
Program: M.Sc. (Physics) **Time : 03 hrs.**
Course Code: PHYS 7014 **Max. Marks: 100**

Instructions:

SECTION A
(*5Q* x *4M=20Marks*)

S. No.		Marks	CO
Q 1	Show that the divergence of a curl ***A*** is always equal to zero, where ***A*** is a vector.	**4**	**CO1**
Q 2	State Cauchy residue theorem.	**4**	**CO1**
Q 3	Prove that $\Gamma(n+1) = n\Gamma n$	**4**	**CO2**
Q 4	Define contravariant and covariant tensors.	**4**	**CO4**
Q 5	If $f = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$ and $= \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$, then show that the multiplication of permutation is not commutative.	**4**	**CO4**

SECTION B
(*4Q* x *10M= 40 Marks*)

S. No.		Marks	CO
Q 6	Team A has probability *2/3* of winning whenever it plays. If A plays four games, find the probability that A wins (i) exactly two games (ii) at least one game.	**10**	**CO2**
Q 7	Use Runga-kutta method to find y(0.2) for the equation $\frac{dy}{dx} = \frac{y-x}{y+x}, y(0) = 1,$ take $h = 0.2.$ **OR** Using finite difference method, find *y(0.25)*, *y(0.5)* and *y(0.75)* satisfying the differential equation $y'' = x - y$, subject to boundary condition *y(0)=0* and *y(1)=2.*	**10**	**CO3**
Q 8	Evaluate $\int_4^{5.2} \log_e x \ dx$ by Simpson's $3/8^{th}$ rule.	**10**	**CO3**
Q 9	Find the Laplace transformation of $sin\, at$.	**10**	**CO4**

SECTION-C **(*2Q* x *20M=40 Marks*)**			
Q 10	Diagonalize the Matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$, and find A^4. **OR** (a) Evaluate the integral $\int_c \frac{z^2+2}{(z-1)(z-2)}\, dz$, where c is $\lvert z \rvert = 2$. (b) Find the first four terms of the *Taylor* series expansion on the complex variable function $f(z) = \frac{(z+1)}{(z-3)(z-4)}$ about $z = 2$.	**20**	**CO1**
Q11	(a) Prove that the orthogonality relation of *Hermite* polynomial in the form $\int_{-\infty}^{+\infty} e^{-x^2} H_m(x) H_n(x) dx = 2^n n! \pi^{\frac{1}{2}} \delta_{mn}$ (b) Hence show that $\int_{-\infty}^{+\infty} e^{-x^2} H_n^2(x) dx = 2^n n! \pi^{\frac{1}{2}}$	**20**	**CO4**