UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, 2021

Programme: B.Sc. (Hons.) Mathematics **Semester: V**
Course Name: Group Theory II **Max. Marks: 100**
Course Code: MATH 3022 **Duration: 3 Hrs.**
No. of page/s: 02

Section A

Attempt all the questions. Each question carries 4 marks.

(Scan and upload)

1.	Let $G = \{(1), (12)(34), (1234)(56), (13)(24), (1432)(56), (56)(13), (14)(23), (24)(56)\}$ Find the stabilizer of 1 and orbit of 1.	**CO3**
2.	How many elements are of order 2 are in $Z_{2000000} \oplus Z_{4000000}$. Generalize.	**CO2**
3.	What is the order of the factor group $\left(Z_{10} \oplus U(10)\right)/\langle(2,9)\rangle$?	**CO5**
4.	Find all Abelian groups (up to isomorphism) of order 360.	**CO1**
5.	Explain why the correspondence $x \to 3x$ from Z_{12} to Z_{10} is not a homomorphism.	**CO2**

SECTION B

Attempt all the questions. Each question carries 10 marks.

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6.	Up to isomorphism, how many additive Abelian groups of order 16 have the property that $t + t + t + t = 0$ for all t in the group?	**CO2**
7.	Suppose that $\varphi: Z_{50} \to Z_{15}$ is a group homomorphism with $\varphi(7) = 6$. a. Determine $\varphi(x)$ b. Determine the kernel of φ c. Determine $\varphi^{-1}(3)$.	**CO2**
8.	Determine how many elements of $Aut(Z_{720})$ have order 6. Also, determine the isomorphism class of $Aut(Z_2 \oplus Z_3 \oplus Z_5)$	**CO1**
9.	Write down the class equation for the symmetric group S_5. **OR** Determine the class equation for non-abelian groups of orders 39 and 55.	**CO3**

SECTION C **Attempt all the questions. Each question carries 20 marks.** **(Scan and upload)**		
10.	State the Sylow Theorem on the existence of a subgroup of prime-power order. Hence proof the theorem by mathematical induction.	**CO4**
11.	a. Let G be a group and $\|G\| = 30$. Show that either Sylow 3-Subgroup or Sylow 5-subgroup is normal in G. b. Let G be a group and $\|G\| = pq$, where p, q are distinct primes, $p < q$ and p does not divide $q - 1$. Show that G is cyclic. **OR** a. Let G be a group and $\|G\| = 30$. Show that either Sylow 3-Subgroup or Sylow 5-subgroup is normal in G. b. Show that there is no simple group of order 216.	**CO5**