Name: **Enrolment No:**	UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

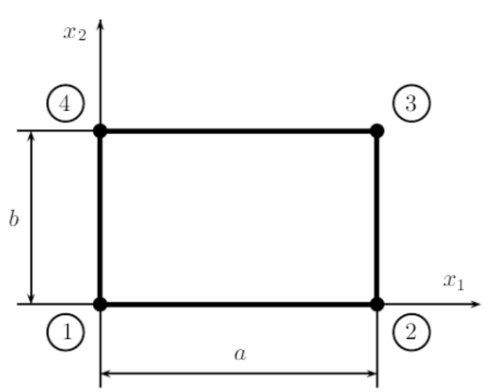
End Semester Examination, December 2021

Course: Solid Mechanics **Semester: V**
Course Code: MECH3022 **Time: 03 hrs.**
Program: BTech- Mechanical **Max. Marks: 100**

SECTION A

S. No.	Question Statement	Marks	CO
Q 1	Explain the properties of Kronecker Delta and Permutation symbol.	4	CO1
Q 2	Explain the summation convention.	4	CO1
Q 3	Describe plane stress and plane strain problems.	4	CO1
Q 4	Describe the types of boundary condition.	4	CO1
Q 5	Explain the properties of influence coefficient.	4	CO1

SECTION B

S. No.	Question Statement	Marks	CO
Q 6	Derive Castigliano's first theorem.	10	CO1
	OR		
	Describe the Maxwell-Betti's reciprocal theorem with proof.		
Q 7	Derive the Beltrami-Michell equation corresponding to isotropic elasticity.	10	CO1
Q 8	A rectangular plate has sides *a* and *b* = 0.6*a*. The plate undergoes deformation by the prescribed displacement field $u_1 = \frac{1}{100}\left(1 - \frac{x_1}{a}\right) x_2$ and $u_2 = \frac{1}{100}\left(1 - \frac{x_2}{b}\right) x_1$. Assume that the deformation of the plate satisfies the plane stress condition and strain components comply with the small strain assumptions. Material is isotropic linearly elastic with Young's modulus E = 20 GPA and Poisson's ratio ν = 0.2. 	10	CO2

	(a) For the prescribed displacement field determine the components e_{11}, e_{22} and e_{12}. (b) Evaluate the strain components at point P with coordinates $x_1 = 0.2a$ and $x_2 = 0.2a$. (c) Determine the stress tensor components σ_{11}, σ_{22} and σ_{12} at point P.		
Q 9	Consider a problem with body forces, $f = \begin{Bmatrix} f_1 \\ f_2 \\ f_3 \end{Bmatrix} = \begin{bmatrix} -6Gx_2x_3 \\ 2Gx_1x_3 \\ 10Gx_1x_2 \end{bmatrix}$ where, $G = \frac{E}{2(1+2\upsilon)}$ and $\nu = \frac{1}{4}$ The displacement field is given as, $u = \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} = \begin{bmatrix} C_1x_1^2x_2x_3 \\ C_2x_1x_2^2x_3 \\ C_3x_1x_2x_3^2 \end{bmatrix}$, determine the constants C_1, C_2 and C_3.	**10**	**CO2**
SECTION-C			
Q 10	(a) Consider a plate with sides 2a and b in x_1 and x_2 directions respectively. The stress distribution is given as $$\sigma_{11} = px_1^3x_2 - 2c_1x_1x_2 + c_2x_2$$ $$\sigma_{22} = px_2^3x_1 - 2px_1^3x_2$$ $$\sigma_{12} = -\frac{3}{2}px_1^2x_2^2 + c_1x_2^2 + \frac{1}{2}px_1^4 + c_3$$ Obtain the corresponding Airy stress function. (b) Show that $\phi = x_1^4x_2 + 4x_1^2x_2^3 - x_2^5$ is a valid Airy stress function, that is, that $\nabla^4\phi = 0$, and compute the stress tensor for this case assuming a state of plane strain with v = 0.25. (c) Determine the constants C_1, C_2, C_3 if the stresses are $\sigma_{xx} = C_1xy$; $\sigma_{yy} = 0$; $\tau_{xy} = C_2 + C_3y^2$; subjected to the boundary, $\tau_{xy} = 0$, at $y = \pm h$; $\int \tau_{xy}\, dy = -P$; $\int \sigma_{xx}\, y dy = -Px$	**20**	**CO3**
Q 11	Derive the expression of Eulerian strain tensor. The strain tensor at a point in a body is given by $\begin{bmatrix} 0.4 & -0.1 & -0.4 \\ -0.1 & 0.2 & 0.3 \\ -0.4 & 0.3 & -0.3 \end{bmatrix} \times 10^{-3}$; Obtain the stress tensor at the point if the body is made of steel with Young's modulus 200 GPa and Poisson's ratio 0.3.	**20**	**CO3**