Name: **Enrolment No:**	UPES UNIVERSITY WITH A PURPOSE

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, DECEMBER 2021

Course: Mathematics-I **Semester: I**
Program: B.Tech. (SOE) **Time: 03 hrs.**
Course Code: MATH 1026 **Max. Marks: 100**
Instructions: All questions are compulsory.

SECTION A (Each question carries 4 marks)

S. No.		**Marks**
Q1	If $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$, find the eigen values of $A^2 - 2A + I$.	**CO1**
Q2	Compute the Jacobian $J\left(\frac{u,v}{x,y}\right)$ if $x = u(1-v), y = uv$.	**CO2**
Q3	If $r^2 = x^2 + y^2 + z^2$, then prove that $\frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2} + \frac{\partial^2 r}{\partial z^2} = \frac{2}{r}$.	**CO2**
Q4	Expand the function $f(\mathrm{x}) = \cos x$ about $x = \frac{\pi}{4}$ in Taylor's series.	**CO4**
Q5	Find the value of a if the vector field $\vec{F} = (2x^2y + yz)\hat{\imath} + (xy^2 - xz^2)\hat{\jmath} + (axyz - 2x^2y^2)\hat{k}$ is solenoidal.	**CO3**

SECTION B (Each question carries 10 marks)

Q1	a. Reduce the quadratic form $Q(x_1, x_2, x_3) = x_1^2 + 2x_2^2 + x_3^2 - 2x_1x_2 + 2x_2x_3$ to the canonical form through an orthogonal transformation. b. Find the rank, signature, index and the nature of this quadratic form.	**CO1**
Q2	Verify **Green's theorem** in $x - y$ plane for $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where C is the boundary of the region bounded by $x = 0, y = 0, x + y = 1$.	**CO3**
Q3	Evaluate the total work done in displacing a particle along the straight line joining the points (0,0,0) to (1,1,1) under the force field $\vec{F} = (3x^2 + 6y)\hat{\imath} - 14yz\hat{\jmath} + 20xz^2\hat{k}$.	**CO3**
Q4	Find the Fourier series for the periodic function $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$ **OR** Find the **half range cosine series** of $f(x) = x,\ 0 \le x \le \pi$.	**CO4**

SECTION-C (This question carries 20 marks)		
Q 1	a. Evaluate $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}}\, dy\, dx$ by **changing the order** of integration. b. If $u = sin^{-1} \frac{x+y}{\sqrt{x}+\sqrt{y}}$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$.	**CO2**
Q 2	Verify **divergence theorem** for $\vec{F} = x^2\hat{\imath} + z\hat{\jmath} + yz\hat{k}$ over the cube formed by the planes $x = -1$, $x = 1, y = -1, y = 1,\ z = -1, z = 1$. **OR** Using **Stoke's theorem** evaluate $\oint_C\ (x+y)dx + (2x-z)dy + (y+z)dz$, where C is the boundary of the triangle with vertices (2,0,0), (0,3,0) and (0,0,6).	**CO3**