UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, May 2021

Programme Name: B.Sc. (Hons.) Mathematics **Semester : VI**
Course Name : Ring Theory and Linear Algebra-II **Time : 3 Hrs**
Course Code : MATH-3023 **Max. Marks : 100**
Nos. of page(s) : 2

Section-A

1. **Each question will carry 5 Marks. 2. Select correct answer in each question. 3. All Questions of this section are compulsory.**

S. No.		CO
Q1	The value of evaluation homomorphism $\phi_2(x^2+x-6)$ where $\phi_2 : Q[x] \to R$ is (a) 0 (b) 2 (c) 4 (d) 8	CO1
Q2	The product of $f(x) = 4x - 5$ and $g(x) = 2x^2 - 4x + 2$ in $Z_8[x]$ is (a) x^2 (b) $6x^2 + 4x + 6$ (c) $8x^3 - 6x^2 + 28x - 10$ (d) $8x^3 - 6x^2 + 28x - 10$	CO1
Q3	Let $\{f_1, f_2, f_3\}$ be the dual basis of $V_3[C]$ with respect to basis {(1,1,1),(1,1,-1),(1,-1,-1)} and $\alpha =$ (0,1,0) then $f_1(\alpha), f_2(\alpha)\ and\ f_3(\alpha)$ are (a) 0, 1 and 2 (b) 1, -1 and 3 (c) 0, ½ and -1/2 (d) 0,0 and 0	CO2
Q4	Let $\phi(x,y) = x - 2y$ be the linear functional on R^2. Transpose $T^t(x,y)$ of the linear operator $T(x,y) = (y, x+y)$ on R^2 is (a) x (b)$-2x - y$ (c)$-x - y$ (d) $< cx, y >= \bar{c} < x, y >$	CO2
Q5	Which one of the following subspace W is not T-invariant in R^3 for given T : $T(a,b,c) = (a+b+c, a+b+c, a+b+c)$ (a) $W = \{(t,t,t) : t \in R\}, T(a,b,c) = (a+b+c, a+b+c, a+b+c)$ (b) $W = \{(x,y,0) : x, y \in R\}, T(a,b,c) = (a+b, b+c, 0)$ (c) $W = \{(0,0,z) : z \in R\}, T(a,b,c) = (a+b, b+c, 0)$ (d) $W = \{(x,y,0) : t \in R\}, T(a,b,c) = (b+c, a+c, a+b)$	CO2
Q6	Which one of the following is not true about an inner product$< x, y >$? (b) $\langle x+z, y\rangle = \langle x,y\rangle + \langle z,y\rangle$ (c) $\langle cx, y\rangle = \bar{c}\langle x,y\rangle$ (d) $\langle x,x\rangle > 0$ for $x \neq 0$. (e) $\langle x,x\rangle = 0$ iff $x = 0$.	CO3

Section-B

1. **Each question will carry 10 Marks. All Questions of this section are compulsory. In Question 5, there is an internal choice.**

S. No.		**CO**
Q1	Prove that $25x^5 - 9x^4 + 3x^2 - 12$ is irreducible over Q using Eisenstein's criterion.	**CO1**
Q2	Prove Gauss's lemma: If D is a UFD, then a product of two primitive polynomials in $D[x]$ is again primitive.	**CO4**
Q3	Verify Cayley Hamilton theorem for a linear operator T on R^2 defined by $$T(a,b) = (a + 2b, -2a + b)$$	**CO2**
Q4	Apply Gram-Schmidt process to orthogonalize subset $\{w_1 = (1,0,1,0), w_2 = (1,1,1,1)\ \ and\ w_3 = (0,1,2,1)\}$ of R^4. Then normalize the vectors to obtain an orthonormal subset.	**CO3**
Q5	Find adjoint $T^*$of the operator $T: C^3 \to C^3$ defined by $$T(x,y,z) = (ix + (2+3i)y, 3x + (3-i)z, (2-5i)y + iz)$$ **OR** Use least square approximation to fit a linear curve on the experimental data (1,2), (2,3), (3,5) and (4,7).	**CO3**

Section-C

1. **The question will carry 20 Marks. 2. Choose one question from two options.**

S. No.		**CO**
Q1	Find all eigenvalues and a basis of each eigenspace of the operator $T: R^3 \to R^3$ defined by $$T(x,y,z) = (2x + y, y - z, 2y + 4z).$$ Also check whether T is diagonalizable or not? Justify your answer. **OR** Let T be a linear operator on $V = P_2(R)$ defined as $T[f(x)] = -xf''(x) + f'(x) + 2f(x)$. Find minimal polynomial of T.	**CO2**