Name: **Enrolment No:**	UPES UNIVERSITY WITH A PURPOSE

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

Online End Semester Examination, May 2021

Course: Riemann Integration & Series of functions **Semester: IV**

Course Code: MATH 2014 **Time: 03 hrs.**

Programme: B.Sc. (Hons.) Mathematics **Max. Marks: 100**

SECTION - A **6 X 5 = 30 Marks**

1. Each Question will carry 5 Marks

2. Instruction: Select the correct option(s)

Q 1	$L(P,f)$ and $U(P,f)$ for the function f defined by $f(x)=x^2$ on $[0,1]$ and $P=\left\{0,\frac{1}{4},\frac{2}{4},\frac{3}{4},1\right\}$ A. $L(P,f)=\frac{7}{32}, U(P,f)=\frac{15}{32}$ B. $L(P,f)=\frac{15}{32}, U(P,f)=\frac{7}{32}$ C. $L(P,f)=\frac{16}{32}, U(P,f)=\frac{7}{32}$ D. $L(P,f)=\frac{5}{32}, U(P,f)=\frac{15}{32}$	CO1
Q 2	A bounded function $f:[a,b]\to$ R is Riemann integrable on $[a,b]$ iff for each $\varepsilon>0$ there exists a partition P of $[a,b]$ such that A. $U(P,\ f)-L(P,f)<\varepsilon$ B. $U(P,\ f)-L(P,f)>\varepsilon$ C. $U(P,\ f)+L(P,f)<\varepsilon$ D. $U(P,\ f)+L(P,f)>\varepsilon$	CO1
Q 3	The integral $\int_0^1 \frac{dx}{x^2}$ is A. Convergent and value is 2 B. Convergent and value is 1 C. Divergent D. Convergent and value is 0	CO2
Q 4	The given series $\sum_{n=1}^{\infty}\frac{n!^2}{2n!}$ converges to A. ¼ B. ½ C. 1 D. 0	CO3
Q 5	The geometric series $\sum_{n=0}^{\infty}(x)^n$ has radius of convergence A. 1 B. -1 C. 0 D. Infinity	CO4
Q 6	The radius of convergence of the following series $$1+\frac{a.b}{1.c}+\frac{a(a+1)b(b+1)}{1.2c(c+1)}+$$ A. 1 B. 1/2 C. 0 D. 3/2	CO4

SECTION – B **10 X 5 = 50 Marks** **1. Each question will carry 10 marks** **2. Instruction: Answer on a separate white sheet, upload the solution as image.**		
Q 1	If $f: [a, b] \to \mathbb{R}$ is a bounded function, then prove that for each $\mathcal{E} > 0$, there exist a $\delta > 0$ such that $U(P, f) < \int_a^{\bar{b}} f(x)dx + \mathcal{E}$ for each $P \in P[a, b]$ with $\|P\| < \delta$.	CO1
Q 2	Prove that if $f \in \mathcal{R}[a, b]$ then $\lvert f\rvert \in \mathcal{R}[a, b]$ and $\left\lvert\int_a^b f(x)dx\right\rvert \leq \int_a^b \lvert f\rvert(x)dx$	CO1
Q 3	Show that the series $$\frac{x}{1+x^2} + \left(\frac{2^2x}{1+2^3x^2} - \frac{x}{1+x^2}\right) + \left(\frac{3^2x}{1+3^3x^2} - \frac{2^2x}{1+2^3x^2}\right) + \cdots$$ does not converge uniformly on closed interval $[0,1]$.	CO4
Q 4	Prove that the integral $\int_a^b \frac{dx}{(x-a)^n}$ is convergent if and only if $n < 1$.	CO3
Q 5	Examine the convergence of the integral $\int_a^\infty \frac{\cos \alpha x - \cos \beta x}{x} dx, \quad a > 0$.	CO3
Section – C **1 X 20 = 20 Marks** **1. Each Question carries 20 Marks.** **2. Instruction: Answer on a separate white sheet, upload the solution as image.**		
Q 1	If a power series $\sum a_n x^n$ converges at the end points $x = R$ of the interval of convergence $[-R, R[$ then prove that it is uniformly convergent in the closed interval $[0, R]$. **OR** If $\sum_{n=0}^{\infty} a_n x^n$ be a power series with finite radius of convergence R, and let $f(x) = \sum_{n=0}^{\infty} a_n x^n, -R < x < R$. If the series $\sum_{n=0}^{\infty} a_n R^n$ converges, then prove that $\lim_{x \to R-0} f(x) = \sum a_n R^n$.	CO4