Name: Enrolment No:	UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, May 2021

Program Name : B. Tech. Appl. Petroleum Engg. Upstream **Semester : IV**
Course Name : Optimization Techniques and Numerical Methods **Time : 03 hrs**
Course Code : MATH 2013 **Max. Marks : 100**
Nos. of page(s) : 02

SECTION A

(Attempt all questions; Each question carries 5 marks)

S. No.		CO
Q1.	After first iteration by using iterative process $x_{n+1} = \frac{1}{2}\left\{x_n + \frac{N}{x_n}\right\}$ the positive square root of 278, with initial solution $x_0 = 16$ is given by A. 16.6800 B. 16.6875 C. 15.6787 D. 17.0989	**CO1**
Q2.	Consider the data $y(15) = 24, y(18) = 37, y(22) = 25$. If using Newton's divided difference formula the second degree polynomial for the above data is given by $p_2(x) = a_0 + a_1(x-15) + a_2(x-15)(x-18)$, then value of a_2 is most nearly A. 24 B. 4.3333 C. -1.0476 D. -3	**CO2**
Q3.	Using three points Simpson's $\frac{1}{3}$ rule an approximate value of the integral $\int_1^2 \frac{\sin \pi x}{\ln x} dx$ is A. 0 B. -2.1678 C. -1.6442 D. -9.8652	**CO3**
Q4.	On the coordinate axes $x = 0$ and $y = 0$, the partial differential equation $x^2 u_{xx} - 2xy\, u_{xy} - 3y^2 u_{yy} + u_y = 0$ is A. Elliptic B. Parabolic C. Hyperbolic D. not classified.	**CO4**
Q5.	The steepest descent direction to minimize the function $f(x_1, x_2, x_3) = 2x_1x_3^2 + x_1x_2x_3$ at the starting point $(1, -1, -1)$ is A. $\begin{pmatrix} -3 \\ 1 \\ 5 \end{pmatrix}$ B. $\begin{pmatrix} 3 \\ -1 \\ 5 \end{pmatrix}$ C. $\begin{pmatrix} 3 \\ 1 \\ -5 \end{pmatrix}$ D. $\begin{pmatrix} 3 \\ 1 \\ 5 \end{pmatrix}$	**CO5**
Q6.	For what values of b the matrix $\begin{bmatrix} 2 & -1 & b \\ -1 & 2 & -1 \\ b & -1 & 2 \end{bmatrix}$ is positive semidefinite? A. $b \le -1$ B. $b \ge 2$ C. $-1 \le b \le 2$ D. $b \in \mathbb{R}$	**CO6**

SECTION B

(Q7-Q10 are compulsory and Q11 has internal choice; Each question carries 10 marks)

Q7. Apply Steepest descent method to minimize the function $f(x_1, x_2) = 4x_1^2 - 4x_1x_2 + 2x_2^2$ with initial point $x_0 = (2,3)$. Perform iterations until $|\nabla f| < \begin{pmatrix}1\\1\end{pmatrix}$. **CO5**

Q8. Using Lagrange multiplier method solve the following constrained optimization problem. **CO6**

$$\min_{x_1, x_2 \geq 0} x_1^2 - x_1x_2 + x_2^2$$

$$\text{subject to } x_1^2 + x_2^2 = 1.$$

Q9. Use Fibonacci search method to minimize the function $f(x) = -\frac{1}{(x-1)^2}\left(\ln x - 2\frac{x-1}{x+1}\right)$ in the range $[1.5, 4.5]$. Reduce the size of the interval minimum $\frac{1}{5}$ of the original. **CO5**

Q10. Evaluate the integral $\int_0^{\frac{\pi}{2}} \sqrt{\sin x}\ dx$ by Simpson's $\frac{3}{8}$ rule with step length $h = \frac{\pi}{12}$. **CO3**

Q11. Estimate the number of students who secured marks between 50 and 55 from the following table. **CO2**

Marks (x)	$30-40$	$40-50$	$50-60$	$60-70$	$70-80$
No. of Students (y)	31	42	51	35	31

OR

Fit a polynomial of degree three, which takes the following values, by Newton forward interpolation formula, and find $y(3.5)$.

x	3	4	5	6
y	6	24	60	120

SECTION C

(Q12a. and Q12b. both have internal choices; Each question carries 10 marks)

Q12. **CO4**

a. Use fourth order Runge-Kutta method to solve for $y(0.4)$ taking $h = 0.2$, for the following initial value problem.

$$\frac{dy}{dx} = 1 + y^2, \text{ with the initial condition } y(0) = \lim_{x\to\infty} \frac{x^2}{2^x}.$$

OR

Using finite difference method determine $y(1.25), y(1.50)$ and $y(1.75)$ for the following boundary value problem

$$x^2y'' + xy' - y = 0 \text{ with } y(1) = \lim_{x\to 1} \frac{\sin(x-1)}{x-1}, y(2) = 0.5.$$

b. Solve the Laplace equation $u_{xx} + u_{yy} = 0$ with $h = \frac{1}{3}$ over the boundary of a square of unit length with $u(x, y) = 16x^2y^2$ on the boundary by Liebmann's iteration process. Perform three iterations of Gauss Siedel method.

OR

Solve $\frac{\partial u}{\partial t} = \frac{1}{2}\frac{\partial^2 u}{\partial x^2}$ with the conditions $u(0,t) = 0, u(4,t) = 0, u(x,0) = x(4-x)$ taking $h = 1$ and employing Bender-Schmidt method. Continue the solution through five time steps.

END