Name:

Enrolment No:

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, May 2021

Course: Ring Theory and Linear Algebra I **Semester: IV**

Programme: B.Sc. (Hons.) Mathematics **Time: 03 hrs.**

Course Code: MATH 2031 **Max. Marks: 100**

SECTION A

Instructions: Attempt all questions. Each question will carry 5 marks.

S. No.	Question	CO
Q1	Let R be the ring of integers under ordinary addition and multiplication. Let R' be the set of all even integers. Let us define addition in R' to be denoted by " $*$ " by the relation $a * b = \frac{ab}{2}$ where ab is the ordinary multiplication of two integers a and b. Then which statement is correct. A. $(R', +, *)$ is a commutative ring. B. R is isomorphic to R'. C. Unit element of R' is 2. D. All are true.	**CO1**
Q2	Which one is not TRUE? A. The set of integers I is only a subring but not an ideal of the ring of rational numbers $(Q, +, \cdot)$. B. The set Q of rational numbers is only a subring but not an ideal of the ring of real numbers $(R, +, \cdot)$. C. If m is a fixed integer, the set P of integers given by $P = \{xm : x \text{ is an integer}\}$ is not an ideal of the ring $(R, +, \cdot)$ of all integers. D. None of the above.	**CO2**
Q3	Consider the real vector space $V = R^3(R)$ and following of its subsets (i) $S = \{(x, y, z) \in V : x = y = 0\}$. (ii) $T = \{(x, y, z) \in V : x = 0\}$. (iii) $W = \{(x, y, z) \in V : z \neq 0\}$. Which one of the following statement is correct A. S, T and W are subspaces. B. Only S and W are subspaces C. Only T and W are subspaces D. Only S and T are subspaces.	**CO4**

Q4	The set $S_1 = \left\{\alpha = \begin{bmatrix} 1 & -2 & 4 \\ 3 & 0 & -1 \end{bmatrix}, \beta = \begin{bmatrix} 2 & -4 & 8 \\ 6 & 0 & -2 \end{bmatrix}\right\}$ and $S_2 = \{f = u^3 + 3u + 4, g = u^3 + 4u + 3\}$ are A. Both linearly dependent B. Both linearly independent C. S_1 is linearly dependent but S_2 is not D. S_2 is linearly dependent but S_1 is not	**CO4**
Q5	If $V(F)$ and $U(F)$ be vector spaces of dimension 4 and 6 respectively. Then $dim\{Hom\ (V, U)\}$ is A. 24 B. 10 C. 6 D. 4	**CO5**
Q6	Consider the mapping (i) $T: R^3 \to R^2, T(x, y, z) = (x + 1, y + z)$. (ii) $T: R^3 \to R, T(x, y) = xy$. (iii) $T: R^3 \to R^2, T(x, y, z) = (\|x\|, 0)$. Which of the above are linear transformation? A. (i) B. (ii) C. (i) and (ii) D. None of the above	**CO5**
SECTION B **Instructions: Attempt all questions. Each question will carry 10 marks. Question 11 has internal choice.**		
Q7	If R is a ring, show that $Z(R) = \{x \in R : xy = yx\ for\ every\ y \in R\}$ is subring of R. Further show that $Z(R)$ is a field if R is a division ring.	**CO1**
Q8	Consider the ring R of all 3×3 matrices of the type $\begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$ under matrix addition and multiplication where a, b, c, d, e, f are real numbers. Show that the set I of all matrices of the form $\begin{bmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is a left ideal of R, which is not a right ideal.	**CO2**
Q9	If R is a ring with unit element 1 and ϕ is a homomorphism of R into an integral domain R' such that the kernel of ϕ i.e. $I(\phi) \neq R$, then prove that $\phi(1)$ is the unit element of R'.	**CO3**
Q10	Find the dimension of subspace of R^4 spanned by the set $$\{(1, 0, 0, 0), (0, 1, 0, 0), (1, 2, 0, 1), (0, 0, 0, 1)\}.$$ Hence, find its basis.	**CO4**

Q11	Let T be a linear operator in R^3 defined by $$T(x_1, x_2, x_3) = (3x_1 + x_3, -2x_1 + x_2, -x_1 + 2x_2 + 4x_3).$$ Find the matrix of T in the ordered basis $\{\alpha_1, \alpha_2, \alpha_3\}$, where $$\alpha_1 = (1, 0, 1), \alpha_2 = (-1, 2, 1), \alpha_3 = (2, 1, 1)$$ **OR** Find a linear transformation $T: R^2 \to R^2$ such that $T(1, 0) = (1, 1)$ and $T(0, 1) = (-1, 2)$. Prove that T maps the square with vertices $(0, 0)$, $(1, 0)$, $(1, 1)$ and $(0, 1)$ into a parallelogram.	**CO5**
SECTION C **Instructions: Attempt all questions. Each question will carry 20 marks. Question 12 has internal choice.**		
Q12	Let U and V be vector spaces over the field F. Let T_1 and T_2 be linear transformations from U into V. The function $T_1 + T_2$ is defined by $$(T_1 + T_2)(\alpha) = T_1(\alpha) + T_2(\alpha) \ \ for\ every\ \alpha \in U$$ is a linear transformation from U into V. If c is any element of F, the function (cT) defined by $$(cT)(\alpha) = cT(\alpha) \ \ for\ every\ \alpha \in U$$ is a linear transformation from U into V. Prove that, the set $L(U, V)$ of all linear transformations from U into V, together with the addition and scalar multiplication defined above is a vector space over the field F. **OR** Prove that, two finite dimensional vector spaces over the same field are isomorphic if and only if they are of the same dimension.	**CO6**