Name:	UPES — UNIVERSITY WITH A PURPOSE
Enrolment No:	

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, June 2021

Course: Real Analysis — **Semester: II**
Program: B. Sc. (Hons.) Maths — **Time : 03 hrs.**
Course Code: MATH 1018 — **Max. Marks: 100**
Instructions: All questions are compulsory.

SECTION A

1. **Each question carries 5 marks.**
2. **Complete the statement / Select the correct answers(s).**

S. No.		
Q1	Which of the following functions is NOT true? a. The empty set is open b. $\mathbb{R}$ is closed c. $\left\{\frac{x}{x+1} : x \geq 0\right\}$ is closed d. $\left\{1 - \frac{x}{x+1} : x \geq 0\right\}$ is not closed	**CO1**
Q2	Let $x \in \mathbb{R}_{>0}$ be some element. Which is FALSE? a. There exists a natural number n such that $n > x$ b. There exists a natural number n such that $n < x$ c. There exist natural numbers m, n such that $n > mx$ d. There exist natural numbers m, n such that $n = mx$	**CO1**
Q3	Consider the set $P = \left\{\frac{1}{m} + \frac{1}{n} : m \in \mathbb{N}, n \in \mathbb{N}\right\}$. Then which is (are) TRUE? a. P is not connected in real line b. P is uncountable c. P is not closed d. P is not dense in real line	**CO1**
Q4	Which of the following is (are) TRUE for a positive term sequence $\{a_n\}$? a. $\lim_{n\to\infty} \left(\frac{a_1 + a_2 + \cdots a_n}{n}\right) = \lim_{n\to\infty} a_n$ b. $\lim_{n\to\infty} (a_1 . a_2 \ldots . a_n)^{1/n} = \lim_{n\to\infty} a_n$ c. $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} (a_n)^{1/n}$ d. $\lim_{n\to\infty} (n)^{1/n} = 0$	**CO2**
Q5	Which of the following is (are) TRUE? a. There exists some $n_0 \in \mathbb{N}$ $s.t. \forall n \geq n_0, \lvert(-1)^n - 1\rvert < \epsilon$ holds b. There exists some $n_0 \in \mathbb{N}$ $s.t.$ $\forall n \geq n_0, \lvert(-1)^n + 1\rvert < \epsilon$ holds c. There exists some $n_0 \in \mathbb{N}$ s.t. $(-1)^n \in N_\epsilon(n_0)$ for infinitely many n d. There exists a unique $n_0 \in \mathbb{N}$ s.t. $(-1)^n \in N_\epsilon(n_0)$ for infinitely many n	**CO2**
Q6	Which of the following is (are) TRUE?	**CO3**

	a. $\sum_{n=0}^{\infty}\frac{(-1)^n}{\sqrt{n}}$ *is absolutely convergent* b. $\sum_{n=0}^{\infty}\frac{(-1)^n}{n^2}$ *is conditionally convergent* c. $\sum_{n=0}^{\infty}\frac{1}{\sqrt{n}}$ *is coditionally convergent* d. $\sum_{n=0}^{\infty}\frac{(-1)^n}{n^{3/2}}$ *is absolutely convergent*	
SECTION B **1. Each question carries 10 marks.** **2. There is an internal choice in Q11.**		
Q7	Prove by giving counterexample that infinite intersection of open sets is not necessarily open.	**CO1**
Q8	Find the limit inferior and limit superior of the following sequence: $$a_n = \left(1-\frac{1}{n}\right)\sin\left(\frac{n\pi}{3}\right)\ , n \geq 1$$	**CO2**
Q9	Consider the sequence $\{a_n\}_{n\geq 1} = \left\{\frac{n!}{n^n}\right\}$. Use Cauchy's theorems on limits to prove that $\lim_{n\to\infty} a_n = 0$.	**CO2**
Q10	Let $\{x_n\}$ be a sequence recursively defined as follows: $$x_1 = 2,\ \ x_{n+1} = \frac{x_n}{2} + \frac{5}{x_n} \ \text{ for } n \geq 1$$ Prove that $\{x_n\}$ converges and find the limit of the sequence.	**CO2**
Q11	Discuss the convergence or divergence of the following series: $$\sum_{n=0}^{\infty}\frac{(n!)^2}{(2n)!}$$ **OR** Discuss the convergence or divergence of the following series: $$\sum_{n=0}^{\infty}\frac{(-1)^n n^2 + n}{n^3+1}$$	**CO3**
SECTION-C **1. Q12 carries (10+10) marks.** **2. There is an internal choice in Q12.**		
Q 12	Consider the infinite series: $$\sum_{n=2}^{\infty}\frac{1}{n\log n}$$ a. Determine whether it is convergent or divergent. b. Use the result of part (a) to determine the convergence of series $$\sum_{n=2}^{\infty}\frac{1}{n[\log(n+1) - \log n]}$$ **OR** Suppose $\{x_n\}$ is a real sequence such that $x_n = \frac{1}{n^\alpha}$ for some $\alpha \in \mathbb{R}$. Prove the following: a. If $\sum_{n=1}^{\infty}\lvert x_n\rvert^p < \infty$ for some $1 < p < \infty$ then $\sum_{n=1}^{\infty}\lvert x_n\rvert^q < \infty$ for any $q > p$. b. If $\sum_{n=1}^{\infty}\lvert x_n\rvert^p < \infty$ for some $1 < q < p < \infty$ then $\sum_{n=1}^{\infty}\lvert x_n\rvert^q = \infty$.	**CO3**