Name:

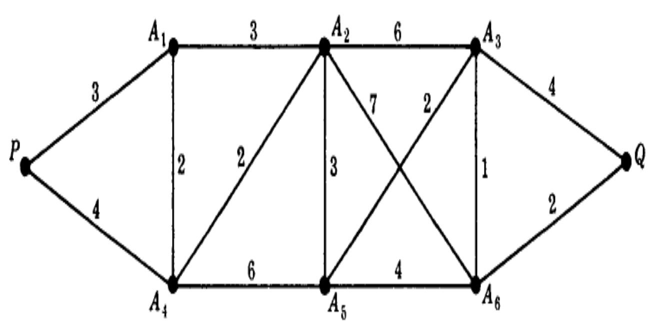
Enrollment No:

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, June 2021

Programme Name: B. Tech. (All SOCS) **Semester : II**
Course Name : Discrete Mathematics **Time : 03 hrs**
Course Code: CSEG 1012 **Max. Marks : 100**

Section A (All questions are compulsory, each question is of 5 marks)		
1.	Let D be a simple graph on 10 vertices such that there is a vertex of degree 1, a vertex of degree 2, a vertex of degree 3, a vertex of degree 4, a vertex of degree 5, a vertex of degree 6, a vertex of degree 7, a vertex of degree 8 and a vertex of degree 9. What can be the degree of the last vertex? A. 4 B. 0 C. 2 D. 5	**CO4**
2.	Radius of a graph *G*, denoted by *rad* (*G*) is defined by....? A. max {$e(v)$: v belongs to V } B. min { $e(v)$: v belongs to V} C. max { $d(u, v)$: u, v belongs to V, u does not equal to v} D. min { $d(u, v)$: u , v belongs to V, u does not equal to v}	**CO4**
3.	In the poset (Z^+, |) (where Z^+ is the set of all positive integers and | is the divison relation) the integers **9** and **351** are … A. comparable B. not comparable C. comparable but not determined D. determined but not comparable	**CO3**
4.	The value of $\boldsymbol{a_4}$ for the recurrence relation $\boldsymbol{a_n = 2\, a_{n-1} + 3}$, with $\boldsymbol{a_0 = 6}$ is….. A. 320 B. 221 C. 141 D. 65	**CO1**
5.	The relation {(1, 1), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2)} on the set {1, 2, 3} is …. A. reflective, symmetric and transitive B. irreflexive, symmetric and transitive C. neither reflective, nor irreflexive but transitive D. irreflexive and antisymmetric	**CO1**
6.	The set {1, i, -i, -1} under the operation multiplication is a … A. semigroup B. subgroup C. cyclic group D. not a cyclic group	**CO5**

	SECTION B **(All questions are compulsory and Q11 has internal choices, each question is of 10 marks)**	
7.	Show that the set of all matrices of the form $\begin{bmatrix} x & x \\ x & x \end{bmatrix}$ where x is non-zero real number is a group under matrix multiplication.	**CO5**
8.	Draw the digraph and the Hasse diagram of $(D_{20}, \leq)$, where $\leq$ is the divisibility relation.	**CO3**
9.	Use a truth table to determine whether the following argument form is valid or not. $p \vee q$ $p \to r$ $q \to r$ $\therefore r$	**CO2**
10.	Show that the relation '*is congruent modulo 4 to*' on the set of integers $\{0,1,2,\ldots,10\}$ is an equivalence relation.	**CO1**
11.	Prove that union of two subgroups of a group G is again a subgroup of G if and only if one is contained in the other. **OR** Let G be a group. If index of a subgroup H in G is two, then prove that H is a normal subgroup of G.	**CO5**
	SECTION C **(Q12 is of 20 marks and it has internal choices)**	
12	If vertices u and v are connected in graph G, the *distance* between u and v in G, denoted by $d(u, v)$, is the length of a shortest (u, v)-path in G; if there is no path connecting u and v we define $d(u, v)$ to be infinite. Show that, for any three vertices u, v and w, $d(u, v) + d(v, w) \geq d(u, w)$. **OR** Check whether the following graph is bipartite, regular, Hamiltonian or not. Using Dijkstra's algorithm, determine the length of the shortest path from ***P*** to ***Q*** 	**CO4**