

***Report***

On

***“DESIGN AND STATIC ANALYSIS OF REAR FUSELAGE  
STRUCTURE”***

*Submitted in Partial fulfillment of the Requirements for the degree of*

**B. Tech. Aerospace Engineering**

By

**Akash Sharma R180206062**

**Vaibhav Bahl R180206059**

**Vatsala Barthwal R180206060**

**Under the Guidance of**

**Mr. Vijay Kumar Patidar**

**Assistant Professor**



Department of Aerospace Engineering

**College of Engineering Studies**

**University of Petroleum & Energy Studies, Dehradun**

**May, 2010**

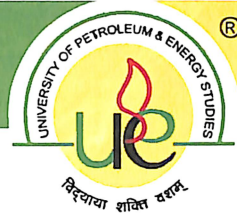
UPES - Library



DI1058

SHA-2010BT

DP-1058  
15/11/11



## UNIVERSITY OF PETROLEUM & ENERGY STUDIES

(ISO 9001:2000 Certified)

### CERTIFICATE

This is to certify that major project report on “**Design and Static Analysis of Rear Fuselage Structure**” submitted to University of Petroleum & Energy Studies, Dehradun; by **Akash Sharma (R180206062)**, **Vaibhav Bahl (R180206059)** and **Vatsala Barthwal (R180206060)**, in partial fulfillment of requirement for award of the degree of B.Tech in Aerospace Engineering is a bonafide work carried out by them under my guidance.

Date:

Place:

Vijay Kumar Patidar

Assistant Professor,

University of Petroleum

& Energy Studies,

Dehradun



## Acknowledgement

We take this opportunity with much pleasure to thank all the people who have helped us in completing our project successfully. Firstly, we sincerely thank our mentor, Mr Vijay Kumar Patidar, for his guidance, help and motivation. Apart from guiding for the project he helped us for choosing the title for the project believed in our ideas and told us ways to implement it.

We would like to express my gratitude to the Faculty of University of Petroleum and Energy Studies who extended their help to us in every possible ways they could. They were always ready and please to help us.

We would like to thank Mr. R.K Tripathi coordinator of B.Tech, Aerospace (University of Petroleum and Energy Studies), and Dr. Om Prakash Head of Dept., Aerospace at University of Petroleum and Energy Studies for their suggestion and support in completion of the project report.



## Abstract

Static analysis is done rear fuselage for the safe and efficient design. Static analysis includes the analysis of frames, stringers, longeron and skin when forces, Moments and torque are applied. It enables the study of stresses, strains and displacement to ensure that these remain below the permissible limits. This is done using ANSYS which is finite element based simulation tool, i.e it uses FEM to find all the stresses induced in the part. Hence the results can be used for product design and optimization which in turn can increase productivity, minimize physical prototyping, reduces the chance of failure, determine the probable cause of it, can be use for non destructive testing and help deliver better and innovative products in less time.

The calculation of stresses caused by Aerodynamic forces and Moments is done on rear fuselage section of baseline aircraft. The initial geometry and shape of the stringers and frame is found by simple structural calculations also best possible option were chosen for the shape of frames and stringers weight and strength being the most prominent criteria. A design is made and hence is modeled using Solidwork11 (CAD software). This model is then analyzed through ANSYS, in which the file is imported and meshed then the calculated loads, moments and torque were applied on it before which the boundary conditions and the material properties were defined. Then the solver was used by which the distribution and intensity of the stresses were found. The results are then shown by color coding the area of same stress by same color and the highly stressed areas are shown by red.

Hence according to results we can determine the whether the dimension for which we have designed the fuselage is correct or not how much stress that design would withstand and what would be the best suited design for the frame and longeron, hence we will accordingly modify the material, geometry and dimension. We can measure the ultimate stress and the failure point. The analysis could be done on the fuselage if crack occurs in it, and see how the distribution of stresses changes accordingly and what would be the failure limit then. This would help in making more efficient design and will also reduce the chance or aircraft accidents.



# CONTENTS

CHAPTER NO.	TITLE	PAGE NO.
	ABSTRACT	iv
	LIST OF FIGURES	viii
	LIST OF TABLE	ix
<b>1.</b>	<b>Introduction to fuselage design</b>	<b>1</b>
	1.1 Fuselage	1
	1.1.1 Truss Type	1
	1.1.2 Monocoque Type	1
	1.1.3 Semi-Monocoque Type	2
	1.2 Primary members in semimonocoque construction	2
	1.3 Designing a fuselage	3
	1.4 Loads acting on Fuselage	4
	1.4.1 Stresses on structure	4
	1.4.2 Types of loads	4
	1.4.3 Design loads	5
	1.4.4 Forces on airplane in flight	5
	1.5 Structural Sizing	6
	1.5.1 Preliminaries	6
	1.5.2 Ways to encounter instabilities	7
	1.5.3 Sectional advantages	9
<b>2.</b>	<b>Rear Fuselage Design</b>	<b>10</b>
	2.1 Introduction to Baseline	10
	2.1.1 Aircraft Data	11
	2.1.2 Specification	12



2.2	Initial calculations	13
2.2.1	Flight envelope	13
2.2.2	Fin lift-curve slope	14
2.2.3	Speed of sound	14
2.3	Balancing out calculations	14
2.4	Fuselage loads	18
2.4.1	Fuselage section	18
2.4.2	Material	19
2.4.3	Weight of rear Fuselage	19
2.4.4	Shear forces and bending moments due to self-weight	20
2.4.5	Total shear forces, bending moments and torques	21
2.5	Fuselage design calculations	23
2.5.1	Stringer sections	23
2.5.2	Skin thickness	28
<b>3.</b>	<b>CAD design and Procedure for analysis</b>	<b>32</b>
3.1	CAD Designing	32
3.2	Analysis	35
3.2.1	Analysis Objective	35
3.2.2	Analysis Procedure	35
3.3	Finite element method	36
3.3.1	Principle	36
3.3.2	Working procedure of Finite element method	36
3.3.3	Advantages of finite element analysis over traditional methods	38
3.4	CAD to Ansys Solving	38
3.4.1	Geometry	38
3.4.2	Material properties	39
3.4.3	Mesh Generation	40
3.4.4	Boundary conditions	41



	3.4.5	Loads	41
	3.4.6	Running the model	41
<b>4.</b>		<b>Results</b>	<b>42</b>
	4.1	Static analysis	42
	4.2	Conclusion	42
<b>5.</b>		<b>Scope of Work</b>	<b>43</b>
		<b>APPENDIX A</b>	<b>44</b>
		<b>APPENDIX B</b>	<b>45</b>
		<b>REFERENCES</b>	<b>46</b>

# List of Figures

Fig 1: Truss type construction of fuselage	1
Fig 2: Monocoque construction	1
Fig 3: Semimonocoque construction with internal members	2
Fig4: A figure showing all the forces and moments with their direction acting on aircraft	5
Fig 5: Instabilities in aircraft component	7
Fig 6: Idealization of frames and skin	8
Fig7: Instability of fuselage	8
Fig 8: Buckling mode of various constructions	8
Fig 9: Aerobatic aircraft taken as baseline configuration	10
Fig 10: Variation of lift and drag coefficient with angle of attack	11
Fig 11: Flight envelop (load factor v/s speed)	13
Fig12: Moment and force acting on wing and tail	15
Fig 13: Stringers arrangement in fuselage cross-section	19
Fig14: Geometrical view of the rear fuselage	20
Fig 15 Positive directions for internal forces and moments	23
Fig 16 Notation and sign convention for forces, moments and displacements	23



Fig 17: Idealized fuselage cross-section	24
Fig18: Placement of Frames in the fuselage	26
Fig19: Stringer section at AA to CC	27
Fig 20: Stringer section at AA to CC	27
Fig21: Fuselage frames Sections	27
Fig 22: Wireframe and solid part view of the rear fuselage modeled in Solidworks	32
Fig 23: Enlargement of a section showing the cross section of frames and stringers	33
Fig 24: Part imported in Ansys	39
Fig 25 Material properties of Aluminum alloy used in the rear fuselage	39
Fig 26 Meshed geometry	40
Fig 27 Applied load on the geometry	41
Fig: 28 Stress results from ansys	42

## **List of tables**

Table 1: Load factor for different aircraft and at different point	13
Table 2: Values of Tail Load found by putting the values of lift in above eq'n	17
Table 3: Values of Tail load, Fin Load and Torque for different loading cases	18
Table 4: Sections wise weight of rear fuselage	20
Table 5: Shear force and moments for different section for different load cases	22



# Chapter 1

## Introduction to fuselage design

**1.1 Fuselage:** Fuselage is the main body of the airplane which carries passengers, cargo, or any payload. The shape for fuselage is cylindrical which has to be streamlined to reduce drag. Generally there are three types of fuselage construction:

**1.1.1 Truss type:** This is a type of construction that uses tubing. Many tubing's are welded together to get a fuselage structure this type of fuselage were used in old navy aircraft and still used in some helicopters.

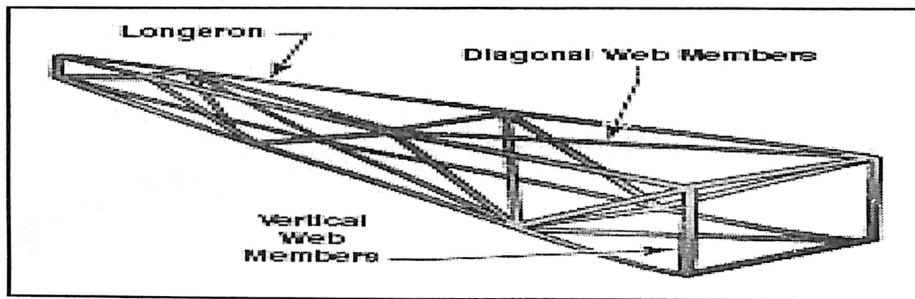


Fig 1: Truss type construction of fuselage

**1.1.2 Monocoque design:** This type of design consist of frame assemblies, and bulkheads to give a proper shape to the fuselage. However, primary stresses are carried by skin No longitudinal members are present for support, so the skin must be strong enough to keep the fuselage rigid. Hence the biggest problem in this construction is maintaining enough strength yet keeping the weight within limits.

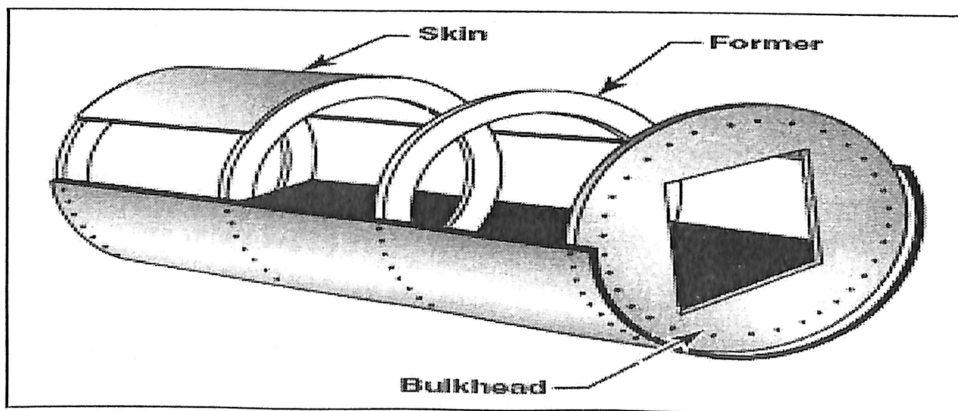


Fig 2: Monocoque construction



**1.1.3 Semi Monocoque Design:** With this design the problem of monocoque construction was solved. In this construction along with frames and bulkhead which are vertical members stringers and longerons were also used these are the horizontal members now the stresses were distributed from the skin to these members hence the thickness of skin could be reduced, which in turn saves a lot of extra weight.

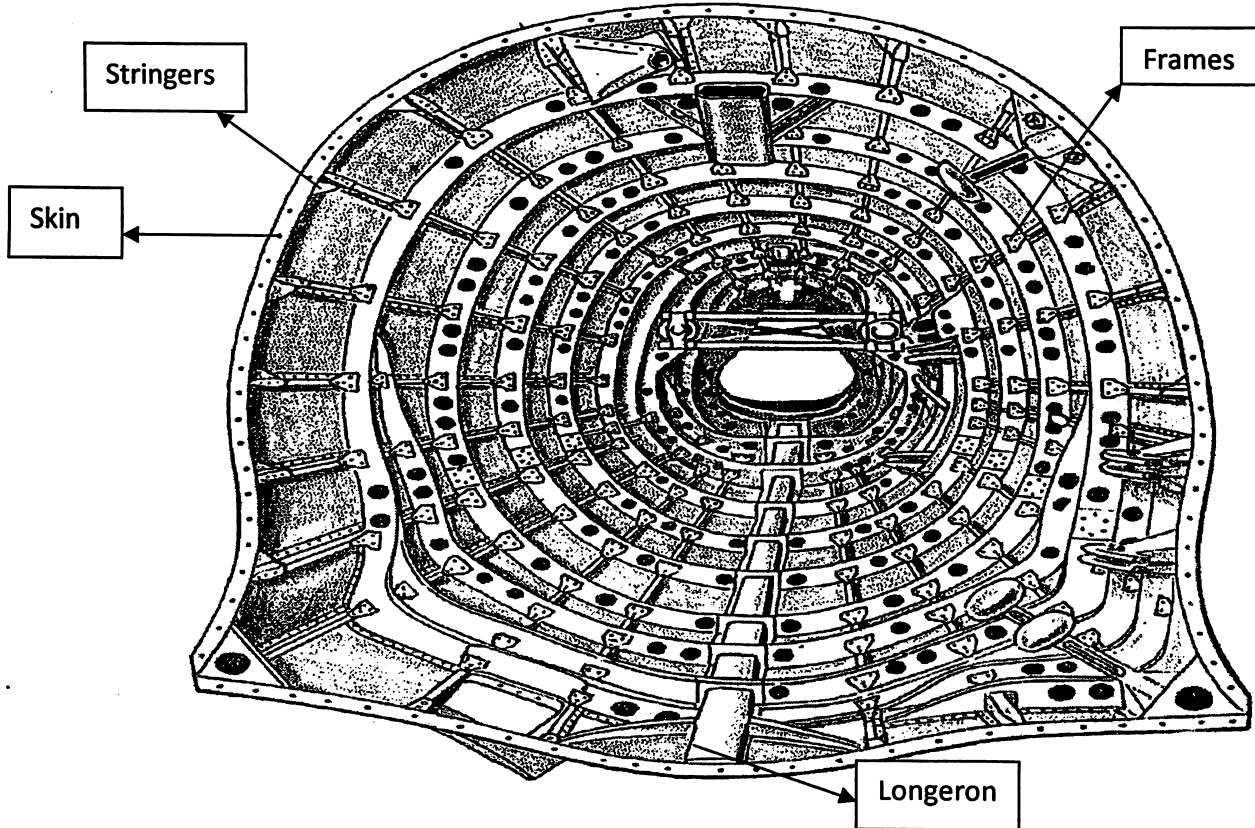


Fig 3: Semimonocoque construction with internal members

## 1.2 Primary members in semimonocoque construction

- **Longerons:** It extent longitudinally at the main points of support are lager in size usually a large fuselage have 4-6 longerons. Primary bending loads are taken by them. Cross sectional shape of longerons are usually I.
- **Stringers:** They are other longitudinal members to supplement longerons. They are much more in number and are light weight than longerons they serve as a fill in's between longerons. A fuselage can have upto 24 stringers or more according to the design. They resist axial loads and divideskin into panels.



- **Frames:** These are vertical members located at some intervals, their main purpose is to provide fuselage desired shape and withstand concentrated load to avoid buckling of fuselage. These members are also found at the attachment points of fitting and other units, like wing-fuselage attachment tail attachments etc. They can be considered as rings but the ring does not have a circular cross-section rather they too have particular sectional shape like C or Z to increase the moment of inertia hence rigidity.
- **Formers:** These are vertical members to supplement frames.
- **Bulkheads:** These are the vertical member that separate two sections from each other for example to separate engine section from fuselage section in low speed propeller aircraft a bulkhead is put in between the two, same is with the radar storage area and fuselage in the fighter plane. We can say that a bulkhead is put at the first and last station of the fuselage. These three vertical members hold the stringers.
- **Skin:** It provide a covering of fuselage, also initially all the loads act upon skin. It is attached to all the other structural members of fuselage. Fuselage skin thickness can vary with the load carried and the stresses occurring at particular area.

### 1.3 Designing a fuselage: Primary considerations before a fuselage is designed are:

- Low aerodynamic drag : fuselage does not contribute largely to lift but increases drag with a very high number so a streamlined designed has to be made although this is the work of aerodynamics group we can say that the fuselage is cylindrical or the formers are circular due to this very reason only.
- Aircraft should have large area to store payload
- Proper and strong structural support for wing and tail. It could handle all forces, moments and torque acting in flight, as well as for landing and ground operation forces.
- Structural optimization to save weight while incorporating protection against corrosion, damage and fatigue.
- The effects that have to be taken care of are: Weight/ strength ratio, material, Joint, Cutouts, Fatigue, Stiffness, Crippling, Yield Strength, Corrosion, Thickness, Cost, Reparability and Maintainability.



## 1.4 Loads acting on Fuselage

Structural integrity is a main consideration in aircraft design and construction. No production airplane will fly before undergoing extensive analysis of how it will behave in flight, the stresses it will tolerate.

Every aircraft is subjected to structural stresses. Stress acts on an airplane in flight as well as in ground. Stress is defined as a load applied to a unit area. Stress produces a deflection or deformation which is called strain. Stress is always accompanied by strain.

Every part of the fuselage should carry a load, or resist a stress appearing on it. Design of supporting and forming pieces, and the skin all have a role to play in assuring an overall safe structure which is capable of withstanding expected loads and stresses.

**1.4.1 Stresses on structure:** Five basic structural stresses to which aircraft are subject are:

1. Tension : It tries to pull things apart
2. Compression : crushing or squeezing force that tries to make part smaller
3. Torsion: It is a twisting force, which resist torque
4. Shear : This stress tries to slide on piece on other, in aircraft the rivets etc are loaded under shear
5. Bending: It is combination of tension and compression. When a part bends inner portion of it undergoes compression and outer tension.

### 1.4.2 Types of loads

An aircraft is required to support these types of basic loads:

- 1) Ground Loads: Encountered by the aircraft during movement on the ground; i.e. taxiing, landing, towing, etc
- 2) Air Loads: Loads exerted on the structure during flight by the maneuvers carried out by the aircraft or by wind gusts.
- 3) Power plant loads: Includes thrust and torque.
- 4) Weight and Inertia loads



### 1.4.3 Design loads:

- **Limit load and ultimate load:** The limit load is the maximum load which the aircraft may encounter at any time during its lifetime, although, yielding/permanent deformation may or may not occur at the limit load. The ultimate load is the load beyond which failure occur. It may be encountered once in the lifetime of an aircraft. All parts must be able to carry this load without failure. Therefore to prevent permanent deformation the material with high ultimate strength is used.
- **Stress Concentrations:** Stress concentrations occur at any discontinuity in the structure discontinuity may be holes, cracks or even change in cross-sectional shape. Stresses at these points are typically high so further more supports are given at these points. The stress concentration factor  $K$  is defined such that  $\sigma_{\max} = K\sigma_{\text{average}}$

### 1.4.4 Forces on airplane in flight

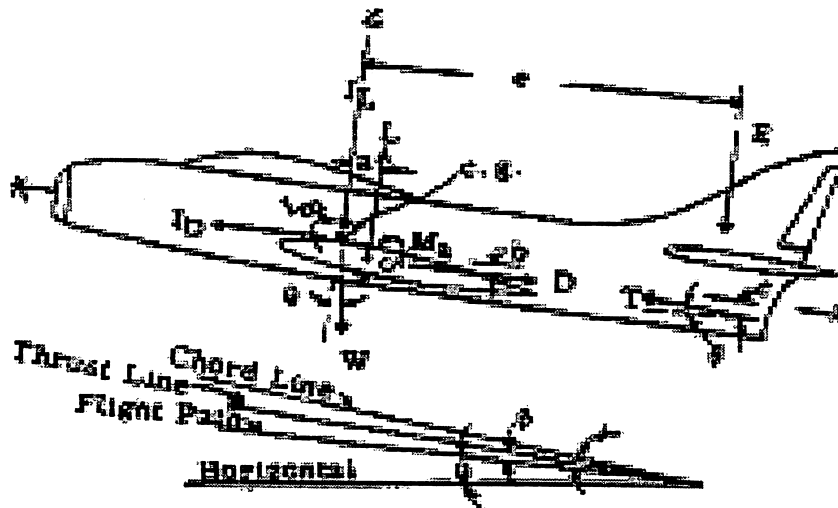


Fig4: A figure showing all the forces and moments with their direction acting on aircraft

Where  $T$  is the thrust produced by engine

$L$  is the total wing lift plus fuselage lift

$D$  is the total airplane drag



$M$  is the moment of Lift and Drag with reference to wing aerodynamic center

$W$  is weight of airplane

$I_L$  is the inertia force normal to flight path

$I_D$  is the inertia force parallel to flight path

$I_M$  is the moment caused by inertia forces

$E$  is tail load normal to the flight

By these forces and moment we can find moment in any point of the fuselage.

## 1.5 Structural Sizing

### 1.5.1 Preliminaries

Before we start designing the fuselage, it is important to know how it is likely to fail. The likely mode of failure is bending and buckling. Before discussing buckling or bending it is important to understand two terms

- Flexural stiffness ( $EI$ ), where  $E$  is Young's modulus and  $I$  is moment of inertia. The bending or flexural stiffness  $EI$  of a beam (or a plate) relates the applied bending moment to the resulting deflection of the beam. We can say that it is resistance to bending.
- Torsional Stiffness ( $GJ$ ) where  $G$  is the shear modulus and  $J$  is the polar moment of inertia. It is a ratio of applied torsional moment to the angle of twist. It is the resistance towards the torsion

In an aircraft Flexural as well as torsional stiffness should be high for increasing Flexural stiffness, Moment of inertia should be increased so we design the frame and longerons with particular cross section such as C, I or Z, flexural stiffness of these section are much more than the plane circular(closed section). Now to prevent torsion, torsional stiffness is increases by





increasing  $J$  i.e. polar moment of inertia which can be increased by using closed section such as circular or rectangular. Therefore circular (ring shaped) frames are made.

If the structure is not stiffened properly then buckling, bending or twisting takes place:

- Buckling: It is the lateral displacement of the beam, panel or plate when a load is applied along its longitudinal axis.
- Bending: It is displacement about lateral axis when a transverse load is applied.
- Twisting: Is the turning of structure about the elastic axis when a tangential force is applied

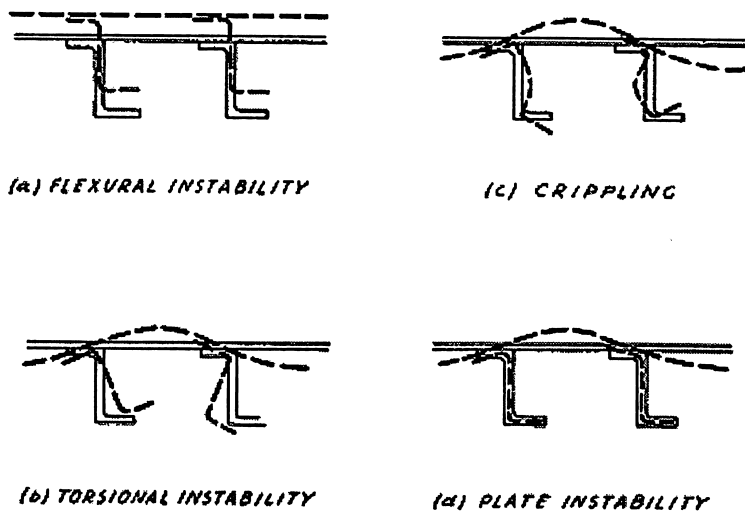
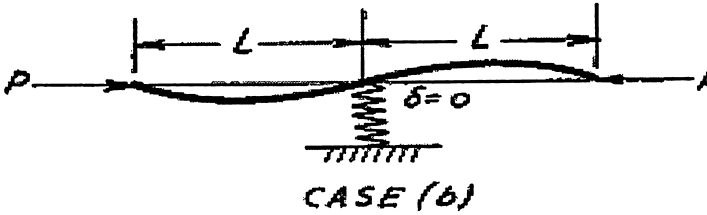
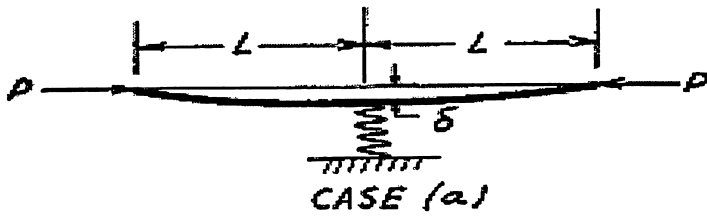


Fig 5: Instabilities in aircraft components

### 1.5.2 Ways to encounter instabilities

In figure below an idealization of the frame and skin-stringer combination is shown. The frames act as springs that resist the displacement of the skin-stringer combination.

In the case where the frames or ribs are themselves not stiff enough, the structure buckles like long rod. But if the frames or ribs are stiff enough, a local buckling occurs. In practice, structures are designed so that local buckling occurs first. The general type of failure is considered to be unacceptable for aerospace structures



Case (a) Weak spring;  
Case (b) Strong spring—effectively rigid.

Fig 6: Idealization of frames and skin

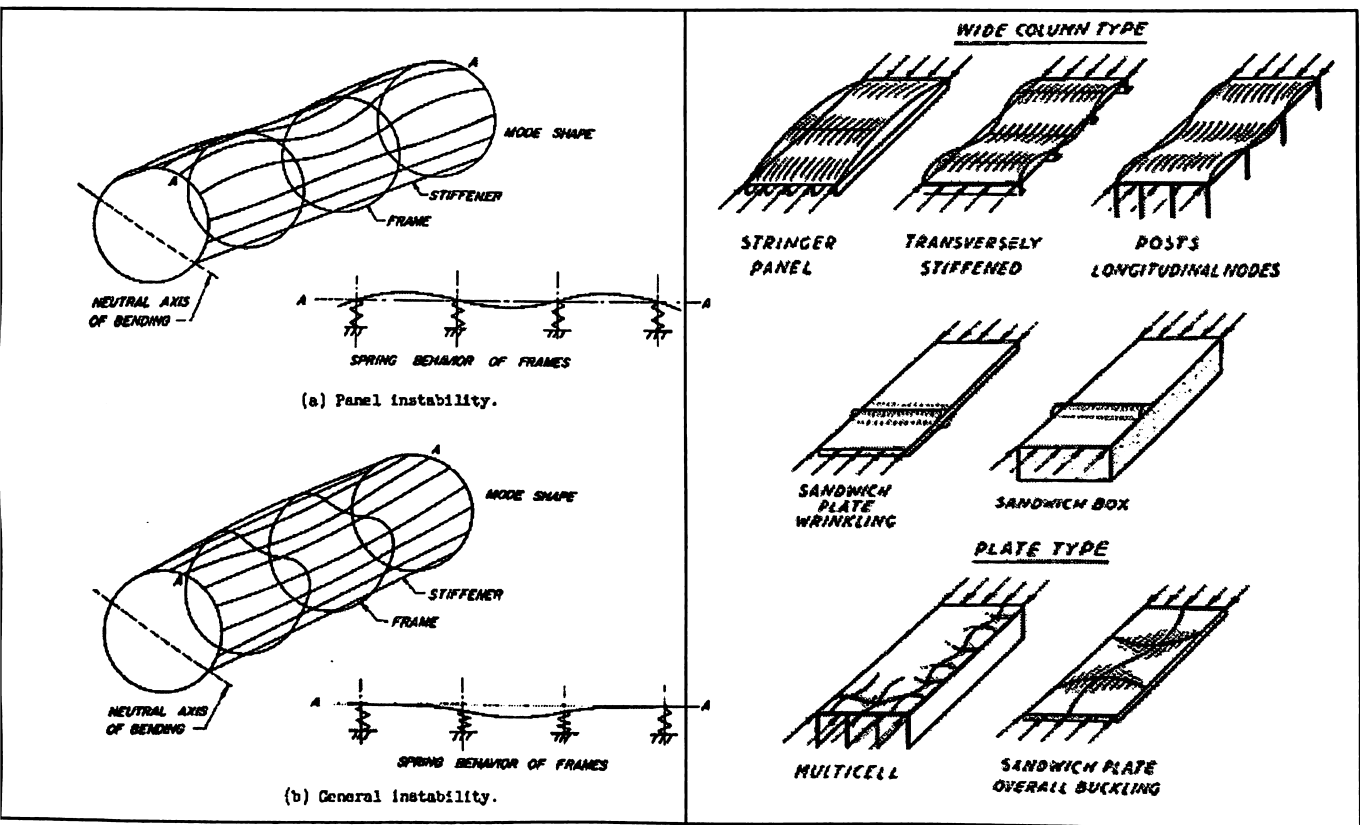


Fig7: Instability of fuselage

Fig 8: Buckling mode of various constructions

Structural designers have to prevent the skin-stringer combination from buckling and to make the frames stiff enough to prevent the global type of failure. This can be done as follows



- **Frames:** The primary purpose of frames is to provide end restraints for the skin and to resist plane deflections when the skin tries to buckle. The goal of frame or rib design is to select a stiffness, that is a material and cross sectional area, and spacing to make the local buckling of the skins as likely as the global buckling of the entire structure (large spacing and low stiffness).
- **Panels:** Panel refer to a composite structure containing skins and stiffeners (stringers). Sketches of panels were shown in the above figures. Panels can be constructed from different types of materials and stringer cross sections. The choice of material depends upon cost and material properties depending on the types of loads the structure will encounter. There are many families of cross sections: I, C, Z, Hat, J, and Y are some section of primary concern.

### **1.5.3 Sectional advantages**

- I section have the highest moment of inertia so used as section for longerons and spars.
- C section are also used for making longerons but at those areas where bending is less. They are not as effective as I section.
- Hat stringers are easy to construct, but when attached to the skin, they create a closed pocket of air that can hide corrosion. This can be overcome by using the hat stringers as fuel vents in wing structures.
- Z stringers are also easily constructed, but have an offset shear center. When deflected, they also twist. They prevent moisture to accumulate.
- J stringers can be hard to form, but are useful for joining 2 panels together.
- Y stringers were studied long ago by NACA. Although hard to form, NACA found that the Y stringer was the best of all of the stiffeners.

**Note: Dimension used for designing a particular section is given in appendix A. and the sectional properties are given in Appendix B**



## Chapter: 2

# Rear Fuselage Design

### 2.1 Introduction to Baseline

The purpose is to design and analysis a two seater trainer/semi-aerobatic aircraft for loads and stresses acting on it. It is required to carry out the detailed structural design layout and analysis of the portion of the rear fuselage between the sections AA and BB. The hand calculations were done for calculating various moments and loads acting on the section. Then to analyses the effect of the loads and moment the fuselage was designed in Solidworks and then the analysis was done using Ansys.

Baseline configuration: A two seater aerobic aircraft is taken as baseline configuration. The side view of the aircraft is shown below

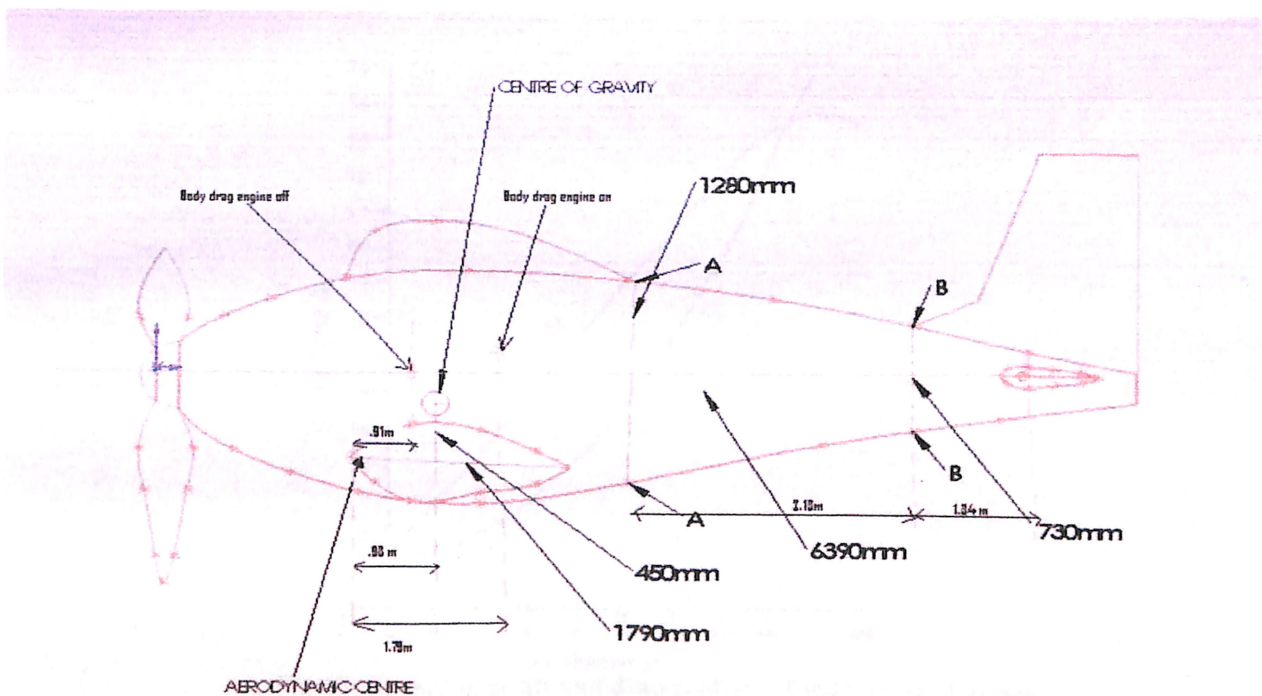


Fig 9: Aerobatic aircraft taken as baseline configuration

Since only dimensions and aircraft data was known a preliminary design was done to visualize how the aircraft will look like, using solid modeling this design was based on the study of configuration of different aerobic aircraft



### 2.1.1 Aircraft Data

The design Data of this aircraft is :

- Fully loaded weight=37.43 kN.
- Moment of inertia of fully loaded aircraft about the centre of gravity  $G = 22\ 235\ \text{kgm}^2$
- The body drag coefficients are  
 $C_{D,B}(\text{engine on}) = 0.01583$   
 $C_{D,B}(\text{engine off}) = 0.0576$
- Maximum horse power of engine =905 and propeller efficiency is 90%.

#### Wing

- Span = 14.07 m
- Gross area = 29.64 m<sup>2</sup>
- Aspect ratio =3 (from historical data)
- Aerodynamic mean chord  $c=2.82\ \text{m}$
- Variations of lift and drag coefficients with incidence are shown in as:

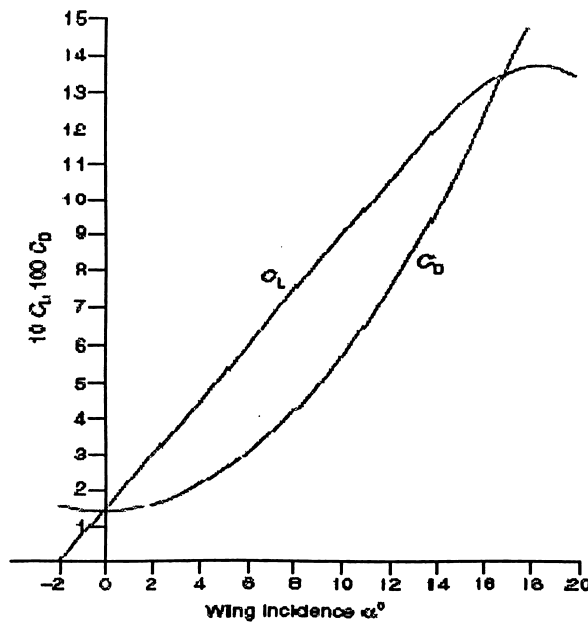


Fig 10: Variation of lift and drag coefficient with angle of attack

- The pitching moment coefficient is given by  $C_M = -0.238C_L$
- Due to a rigger's incidence of  $-1.5^\circ$ , Additional pitching moment coefficient equal to  $-0.036$ .

**Tailplane**

- Span = 6.55 m
- Gross area = 8.59 m<sup>2</sup>
- Position of the aerodynamic centre, P, of the tailplane is shown in figure of aircraft.
- Asymmetric loads are induced on the tailplane, due to the asymmetry of the slipstream induced by yaw. These loads, upwards on one side and downwards on the other, result in a torque given which is given by:

$$\frac{0.00125}{\sqrt{1 - M^2}} \rho V^2 S_t b_t \psi$$

where  $M$  is the mach number,

$S_t$  the tailplane area

$b_t$  the tailplane span.

**Fin**

- Height = 1.65 m
- Area = 1.80 m<sup>2</sup>
- Aspect ratio = 1.5.
- Lift-curve slope,  $dC_l/d\alpha$ , of the fin is given by

$$\frac{dC_l}{d\alpha} = \frac{5.5A}{A + 2}$$

Here  $A$  is the aspect ratio of a wing which is twice the aspect ratio of fin.

- In a yawed flight the incidence of the fin to the air flow is  $\psi$  hence the fin load is equal to  $(\frac{1}{2})\rho(V^2)S_F\left(\frac{dC_l}{d\alpha}\right)\psi$  where  $V$  is the aircraft speed and  $S_F$  the fin area.
- By geometry of pressure distribution we can find the center of gravity for the fin
- Centre of pressure is found to be 1.13m above the axis of the rear fuselage and at a distance of 3.7m aft of the section AA.



### 2.1.2 Specification

The flight envelope for an aircraft is:

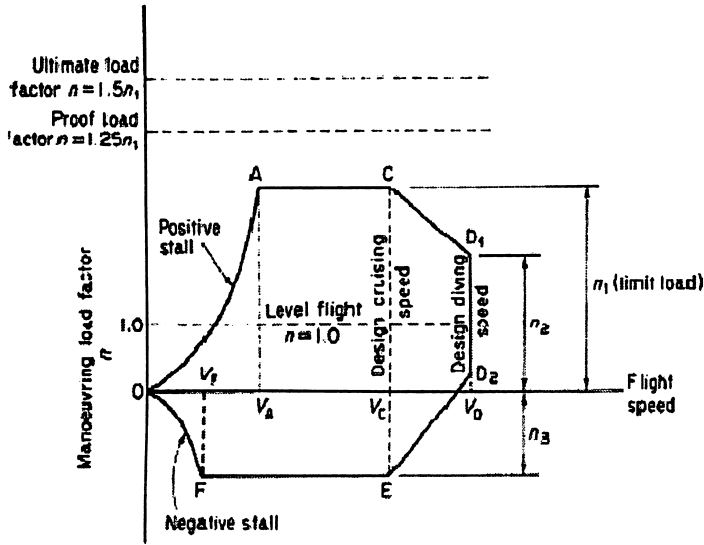


Fig 11: Flight envelop (load factor v/s speed)

Load factor	Normal	Semi Aerobatic	Aerobatic
$n_1$	$2.1+24000(W+10000)$	4.5	6
$n_2$	$0.75n_1$ but $n_2$ not less than 2	3.1	4.5
$n_3$	1	1.8	3

Table 1: Load factor for different aircraft and at different point

For an aerobatic aircraft  $n_1 = 6.28$ ,  $V_D$  (design diving speed) = 183.8m/s

Also,

Cruising speed  $V_c$  is given by:

$$V_C = 0.8V_D = 147.0\text{m/s}$$

$$n_2 = 0.75n_1 = 4.71$$

$$n_3 = 0.5n_1 = 3.14$$

Note that the airworthiness requirements specify that since  $n_1 > 3$  the point  $D_2$  lies on the  $n=0$  axis.

Further requirements are:

- (i) At any point on the flight envelope an additional pitching acceleration given by:

$$\frac{(20 + \frac{475}{W})n}{V} \text{ rad/s}^2$$



Where  $W$  is the total weight of the aircraft in kN and  $V$  is the velocity of the aircraft in m/s.

(ii) For asymmetric flight an yaw angle allowed for the aircraft is given by given by:

$$\psi = 0.7n1 + \frac{457.2}{Vd} \text{degrees}$$

The angle of yaw increases the overall pitching moment coefficient of the aircraft by  $-0.0015/\text{degree}$  of yaw.

## 2.2 Initial calculations

### 2.2.1 Flight envelope

The maximum coefficient of lift for a steady straight level flight is given by:

$$C_{L, max} = \frac{nW}{\left(\frac{1}{2}\right)\rho V_s^2 S}$$

where  $V_s$  is the stalling speed

$S$  the wing area.

Then

$$V_s = \left[ \frac{2nW}{\rho S C_{Lmax}} \right]^{\frac{1}{2}}$$

Substituting the values in this equation from the given aircraft data and taking  $\rho$ , the density of air at sealevel=  $1.226 \text{ kg/m}^3$

$$V_s = \left[ \frac{(2 * 37.43 * 10^3)}{1.226 * 29.64 * 1.38} \right]^{\frac{1}{2}} * n^{\frac{1}{2}}$$

$$V_s = 38.6(n)^{\frac{1}{2}}$$

The positive stall curve is found by giving a series of values to  $n$  and then calculating the corresponding stalling speeds. We have found load factor to be 6.28 so for  $n=n_1=6.28$

Putting value of  $n$  in above equation we get

$$\begin{aligned} V_s &= 38.6(6.28)^{1/2} \\ &= 96.7 \text{ m/s (point A on flight envelope)} \end{aligned}$$





## 2.2.2 Fin lift-curve slope

- $dCl/d\alpha = (5.5A)/(A + 2)$
- $dCl/d\alpha = (5.5 * 3.0)/(3.0 + 2.0) = 3.3$

## 2.2.3 Speed of sound

Speed of sound is 340.8 m/s at sea level and a temperature of 15°C.

## 2.3 Balancing out calculations

The tailplane and fin loads corresponding to the various important points on the flight envelope will be calculated so that, subsequently, values of shear force, bending moment and torque acting on the rear fuselage due to these loads may be determined. The cases to be investigated are:

- Case A (point A on the flight envelope with engine on)
- Case A (point A on the flight envelope with engine off)
- Case C (point C on the flight envelope with engine off)
- Case D1 (point D1 on the flight envelope with engine off)
- Case D2 (point D2 on the flight envelope with engine off)

### Case A

From the flight envelope  $n=6.28$ ,  $V=96.7$  m/s and from fig 10 the wing incidence  $\alpha$  which corresponds to  $C_{L,max}$  is at  $18^\circ$  and value of  $C_{L,max}$  is 1.38 The forces acting on the aircraft and their lines of action are shown in Fig. 12

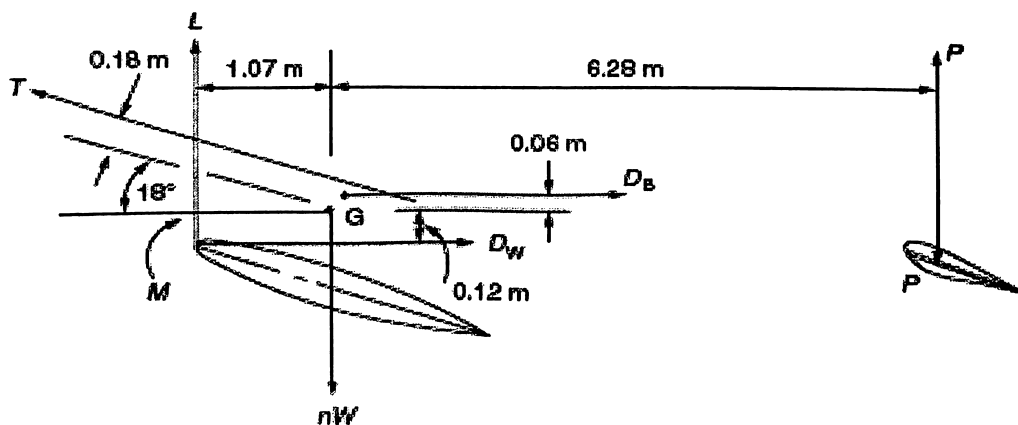


Fig12: Moment and force acting on wing and tail



Since  $1 \text{ hp} = 746 \text{ W} = 746 \text{ mN/s}$ , the thrust  $T$  of the engine is given by

$$T = \frac{\eta * hp * 746}{V}$$

i.e.  $T = (0.9 * 905 * 746) / 96.7$

$$T = 6284 \text{ N}$$

Also,  $nW = 6.28 * 37.43 * 10^3 = 235060 \text{ N}$

First approximation for the wing lift,  $L$ , is obtained by neglecting the tailplane lift,  $P$ , i.e.

$$L = nW - T \sin(18^\circ - 1.5^\circ)$$

which gives

$$L = 235060 - 6284 \sin 16.5^\circ = 233275 \text{ N}$$

From Fig10. the wing drag coefficient,  $C_{D,W}$ , is 0.149 so that the wing drag, which is given by

$$D_W = C_{D,W} \left(\frac{1}{2}\right) \rho (V^2) s$$

is

$$D_W = 0.149 * 1.226 * 96.7^2 * 29.64 / 2 = 25315 \text{ N}$$

The body drag coefficient is 0.01583 so that

$$D_B = 0.01583 * 1.226 * 96.72 * 29.64 / 2 = 2690 \text{ N}$$

The angle of yaw is given by

$$\psi = 0.7n1 + 457.2/Vd \text{ degrees}$$

$$\psi = (0.7 * 6.28) + (457.2 / 183.8) = 6.9^\circ$$

The total pitching moment coefficient is then

$$C_M = (-0.238 * 1.38) + (-0.036) + (-0.0015 * 6.9) = -0.375$$

So that  $M = C_M * \left(\frac{1}{2}\right) \rho (V^2) S c = -0.375 * 1.226 * 96.7^2 * 29.642.82 / 2 = -179 669 \text{ Nm}$

The additional pitching moment acceleration is,

$$\frac{\left(20 + \frac{475}{W}\right) n}{V} \text{ rad/s}^2$$

$$\frac{\left[\left(20 + \frac{475}{37.43}\right) 6.28\right]}{96.7} = 2.12 \text{ rad/s}^2$$



Now, taking moments about G from fig 12

$$1.07L - 0.18T + 0.06D_B - 0.12D_W - 6.28P - 179669 = 22235 * 2.12$$

i.e.  $1.07L - (0.18 * 6284) + (0.06 * 2690) - (0.12 * 25315) - (6.28P) - 179669 = 22235 * 2.12$

This simplifies to:

$$5.78P = L - 215715$$

First approximation	L = 233275 N	P = 2991 N
Second approximation	L = 233275 - 2991 = 230284 N	P = 2482 N
Third approximation	L = 233275 - 2482 = 230793 N	P = 2569 N
Fourth approximation	L = 233275 - 2569 = 230706 N	P = 2554 N
Fifth approximation	L = 233 275 - 2554 = 230 721 N	P = 2556 N

Table2: Values of Tail Load found by putting the values of lift in above eq'n

After fifth approximation it was found that the tail load  $P = 2556$  N. (as it was not varying much after the fourth approximation)

- The torque produced by the asymmetric loading on the tailplane is given by

$$\frac{0.00125}{\sqrt{1 - M^2}} \rho V^2 S t b t \psi$$

i.e. Tailplane torque =  $[0.00125 / [1 - (96.7/340.8)^2]^{1/2}] * 1.226 * 96.7^2 * 8.59 * 6.55 * 6.9 = 5802$  Nm

- The load on the fin caused by the yawed flight is given by  $1/2 \rho V^2 S \psi A$ , i.e.

$$\text{Fin load} = 1.226 * 96.7^2 * 1.8 (6.9 * \pi / 180) * 3.3 / 2 = 4100 \text{ N}$$

The torque produced on the fuselage by this fin load is  $4100 * 0.13 = 4633$  Nm.

- The total torque on the rear fuselage is therefore given by:

$$\text{Total torque (rear fuselage)} = 5802 + 4633 = 10435 \text{ Nm}$$

The tail and fin loads and the rear fuselage torque corresponding to the remaining flight envelope cases are calculated in an identical manner and are listed in Table below



Case	Tail load (N) (+ ↑)	Fin load (N) (+→)	Fuselage Torque(N m) (+ ↓)
A	2556	4100	10435
A'	2292	4100	10435
C	596	9501	24957
D <sub>1</sub>	4997	12460	34031
D <sub>2</sub>	9412	5340	14635

**Table3: Values of Tail load, Fin Load and Torque for different loading cases**

## 2.4 Fuselage loads

The portion of rear fuselage which is to be designed and analyzed is portion from AA to BB in fig 9

### 2.4.1 Fuselage section

The construction of structural components was discussed in chapter 1 where it was discussed that fuselages generally comprise arrangements of stringers, frames and skin. Our baseline configuration is an aerobatic aircraft hence the fuselage is unpressurized this means that the frames will not support significant loads. But they will be required to maintain a proper fuselage shape and therefore may be nominal in size, suitable frame sections will be designed in the later sections of the chapter. The combination of skin and stringers will resist the shear forces, bending moments and torques produced by aerodynamic loads and self weight. Also keeping in mind all the rigidity requirements a circular cross-section is chosen that will meet the design requirements of the aircraft and will be simple to fabricate and design. Stringers will be made of Z section as they provide a descent support for less loaded structure the schematic diagram of arrangement of stringers in frame is shown in the figure below afterwards these stringers will be considered as booms i.e circular section hence the area of booms will be found out from which designing of the Z section will be done.

**Fig 13: Stringers arrangement in fuselage cross-section**

This Figure shows a possible section. Twenty-four stringers arranged symmetrically, each having the same cross-sectional area and would be spaced at approximately 168mm at the section AA and at 96mm at the section BB.

### 2.4.2 Material

An aluminum alloy is used for both stringers and skin because of light weight and several other useful properties as:

- 0.1% Proof stress = 186N/mm<sup>2</sup>
- Shear strength = 117N/mm<sup>2</sup>
- Young's modulus=7.1 \*10<sup>10</sup> pa
- Poisons ratio= .33

### 2.4.3 Weight of rear Fuselage

In a single-engine aircraft as our baseline configuration shown in Fig.9 it is assumed that the fuselage weight is 4.8% to 8.0% of the total weight and that the weight of the tailplane/fin assembly is 1.2% to 2.5% of the total weight of the aircraft. In this case it is further assumed that half of the fuselage weight is aft of the section AA and the weight distribution varies directly as the skin surface area. Therefore, taking average values

- Weight of rear fuselage =  $(37.43 * 10^3 * 6.4) / (2 * 100) = 1198N$
- Weight of tail plane/fin =  $(37.43 * 10^3 * 1.8) / 100 = 674N$

For calculation the rear fuselage is assumed to have a uniform taper as shown in Fig 14; CC is a section midway between AA and BB.

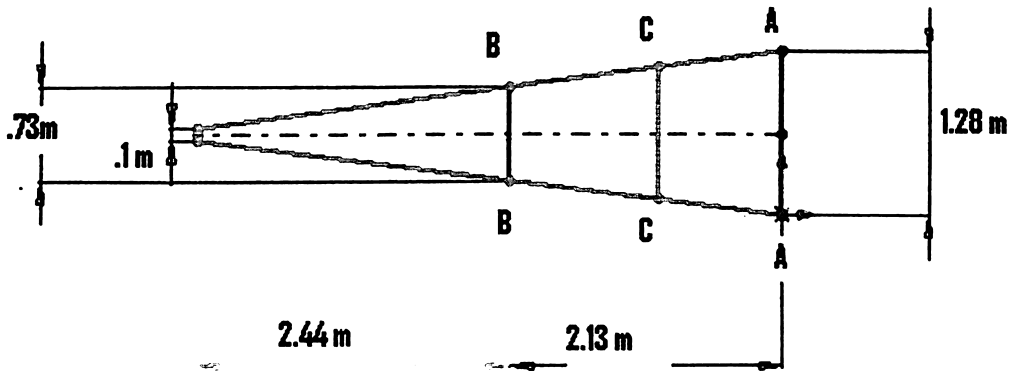


Fig14: Geometrical view of the rear fuselage

The total skin area of the rear fuselage is given by:

$$\text{Skin area} = (\pi (1.28 + 0.1) \times 4.57) / 2 = 9.91\text{m}^2$$

Section	Weight/m of fuselage (N/m)
AA	$1198 \times \pi \times 1.28 / 9.91 = 486.1$
CC	$1198 \times \pi \times 1.01 / 9.91 = 383.6$
BB	$1198 \times \pi \times 0.73 / 9.91 = 277.2$
DD	$1198 \times \pi \times 0.1 / 9.91 = 38.0 \text{ N}$

Table4: Sections wise weight of rear fuselage

Also it is estimated that the centre of gravity of the tail plane/fin assembly is 4.06m from the section AA on a line parallel to the fuselage centre line.

#### 2.4.4 Shear forces and bending moments due to self-weight

At the section AA

- $SF = (1198 + 674) * n = 1872n \text{ N}$
- $BM = [(38.0 * 4.57^2 / 2) + (448.1 * 4.57^2 / 2 * 3) + (674 * 4.06)]n \cos \alpha = 4693n \cos \alpha \text{ Nm}$

Where n is the normal acceleration coefficient and  $\alpha$  wing incidence.

**At the section CC**

- $SF = [1872 - (486.1 + 383.6) * (2.13/2 * 2)] n = 1409n \text{ N}$
- $BM = [(38.0 * 3.51^2/2) + (345.6 * 3.51^2/2 * 3) + (674 * 2.99)] n \cos \alpha = 2959n \cos \alpha \text{ Nm}$

**At the section BB**

- $SF = [1872 - (486.1 + 277.2) * (2.13/2)] n = 10059n \text{ N}$
- $BM = [(38.0 * 2.44^2/2) + (239.2 * 2.44^2/2 * 3) + (674 * 1.93)] n \cos \alpha = 1651n \cos \alpha \text{ Nm}$

**2.4.5 Total shear forces, bending moments and torques**

The values of shear force, bending moment and torque at the sections AA, BB and CC will be calculated for all the flight envelope cases.

**Case A** ( $n=6.28$ ,  $\alpha=18^\circ$ )

**Section AA**

Shear force due to the self-weight (Inertia force) and tail load is, from  $1872n$  and Table 3

- $SF (S_y) = (1872 \times 6.28) - 2556 = 9200 \text{ N}$  (acting vertically downwards)

The shear force due to the fin load is, from Table 3

- $SF (S_x) = 4100 \text{ N}$  (acting horizontally towards right)

The bending moment due to the self-weight and tail load is, from  $BM=4693n \cos \alpha \text{ Nm}$  and Table 3

- $BM (M_x) = 4693 \times 6.28 \cos 18^\circ - 2556 \times 3.47 = 19160 \text{ Nm}$

The bending moment due to the fin load is, from Table 3

- $BM (M_y) = 4100 \times 3.7 = 15170 \text{ Nm}$

The torque due to asymmetric flight and the fin load is, from Table 3

- $T = 10435 \text{ Nm}$

The values of shear force, bending moment and torque at the section AA due to the remaining flight envelope cases are calculated in similar manner. The complete procedure is then repeated for the sections CC and BB. The results are listed in Table 5 with the positive direction's and senses of forces, moments and torques are as shown in Fig. 15; these are for an internal section when viewed in the perpendicular direction Oz (Fig. 16) except for torsion where it is assumed that a positive fin load produces a positive torque.

**Section AA**

Case	$S_x$ (N)	$S_y$ (N)	$M_x$ (N m)	$M_y$ (N m)	T (N m)
A	4100	9200	19160	15170	10435
A'	4100	9434	19938	15170	10435
C	9501	11125	27534	28958	24957
D <sub>1</sub>	12460	13350	39470	37978	34031
D <sub>2</sub>	5340	9412	32688	16276	14635

**Section CC**

Case	$S_x$ (N)	$S_y$ (N)	$M_x$ (N m)	$M_y$ (N m)	T (N m)
A	4100	6675	10308	8301	10435
A'	4100	7120	11393	8301	10435
C	9501	8811	16276	26490	24957
D <sub>1</sub>	12460	11837	26490	24686	34031
D <sub>2</sub>	5340	9412	22651	10580	14635

**Section BB**

Case	$S_x$ (N)	$S_y$ (N)	$M_x$ (N m)	$M_y$ (N m)	T (N m)
A	4100	4673	4747	3824	10435
A'	4100	5118	5358	3824	10435
C	9501	6809	8003	8687	24957
D <sub>1</sub>	12460	10547	13347	11393	34031
D <sub>2</sub>	5340	9412	12614	4883	14635

**Table:5 Shear force and moments for different section for different load cases**



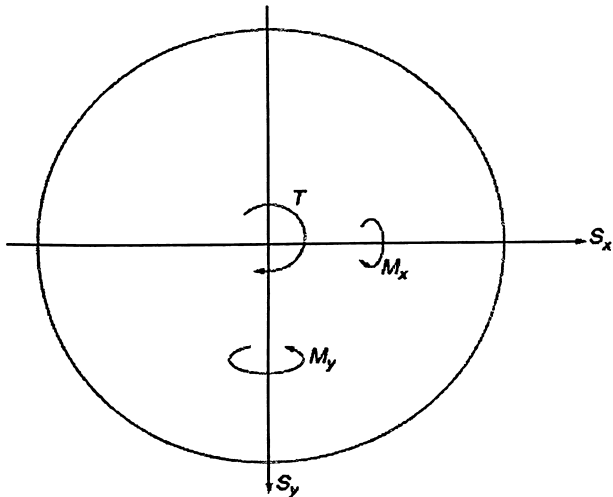


Fig 15 Positive directions for internal forces and moments.

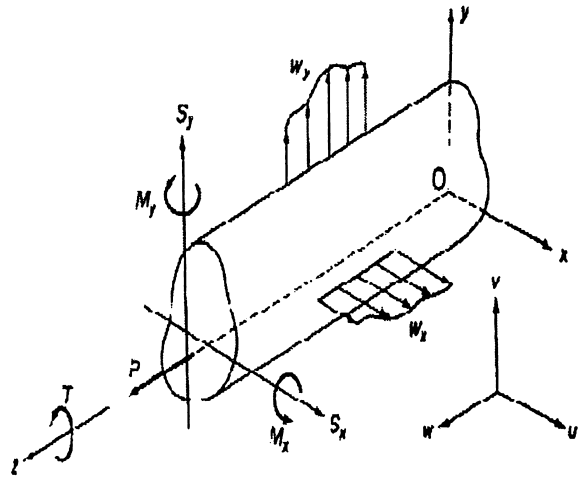


Fig 16 Notation and sign convention for forces, moments and displacements

## 2.5 Fuselage design calculations

Two approaches can be used for actual design. Elastic design uses allowable or working stresses which can be obtained from, 0.1% proof stress and incorporating a factor of safety, usually taken as 1.5; these stresses are then used to calculate skin and stringer sizes. Alternatively, ultimate load design is done by multiplying the actual loads by an ultimate load factor hence we get the failure loads, and the stresses involved are therefore the ultimate stresses. The working, or allowable, stresses are then

- Direct =  $186 / 1.5 = 124\text{N/mm}^2$
- Shear =  $117 / 1.5 = 78\text{N/mm}^2$

The proposed fuselage section is circular as previously shown in Fig13. The design process is required to produce suitable stringer sections and a skin thickness. Now each stringer is supposed to be a boom having cross-sectional area  $B \text{ mm}^2$  and that the skin thickness is  $t \text{ mm}$ . The idealized fuselage section is shown in Fig 17

### 2.5.1 Stringer sections

The direct stress in each stringer produced by bending moments  $M_x$  and  $M_y$  is given by:

$$\sigma_z = \left( \frac{M_x}{I_{xx}} \right) * y + \left( \frac{M_y}{I_{yy}} \right) * x$$

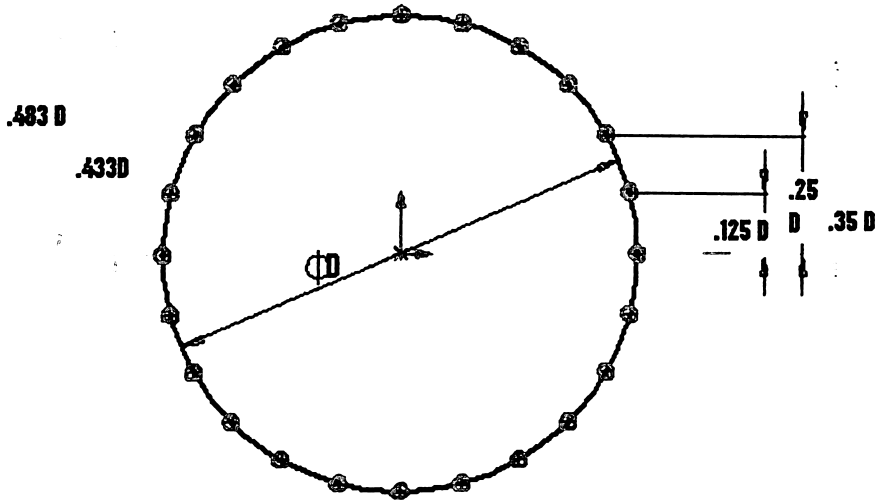


Fig 17: Idealized fuselage cross-section.

Where  $I_{xx} = I_{yy} = 4BD^2(0.1294^2 + 0.25^2 + 0.353^2 + 0.433^2 + 0.483^2 + 0.5^2/2)$

i.e.  $I_{xx} = I_{yy} = 3.0BD^2 \text{ mm}^4$

A positive value of  $M_x$  will cause tensile stresses in stringers 2 to 12 (numbering starts from middle boom right hand side) there will be zero stress in stringers 1 and 13 and compressive stresses in stringers 14 to 24. A positive  $M_y$  produces tensile stresses in stringers 8 to 18 and compressive stresses in stringers 6 to 20, zero stress in stringers 7 and 19. Therefore  $M_x$  and  $M_y$  both produce tensile stresses in stringers 7 to 13 and compressive stresses in stringers 19 to 1; in the remaining stringers the stresses due to  $M_x$  and  $M_y$  are of opposite sign.

We can see in of Table.5 that  $M_x$  and  $M_y$  reach their greatest values at each fuselage section in load Case D1. We will take the maximum value of stresses and load factor according to D1

**Section AA (diameter D=1.28 m)**

$M_x = 39470\text{Nm}$       $M_y = 37978\text{Nm}$

Tensile stresses on the stringers due to  $M_x$  and  $M_y$  are:



$$\sigma_z = \left(\frac{M_x}{I_{xx}}\right) * y + \left(\frac{M_y}{I_{yy}}\right) * x$$

$$\sigma = \left[ \frac{39\,470 * 10^3}{3.0B * 1.28^2 * 10^6} \right] y + \left[ \frac{37\,978 * 10^3}{3.0B * 1.28^2 * 10^6} \right] x$$

i.e.

$$\sigma = (8.03y + 7.73x)/B$$

At stringer 7,  $x=0$ ,  $y=0.64$ m

Then  $\sigma_7 = 8.03 \times 0.64 \times 10^3/B$

i.e.  $\sigma_7 = 5139/B \text{ N/mm}^2$

Similarly  $\sigma_8 = 6245/B \text{ N/mm}^2$

$\sigma_9 = 6924/B \text{ N/mm}^2$

$\sigma_{10} = 7121/B \text{ N/mm}^2$

$\sigma_{11} = 6854/B \text{ N/mm}^2$

$\sigma_{12} = 6109/B \text{ N/mm}^2$

$\sigma_{13} = 4947/B \text{ N/mm}^2$

### Section CC (diameter $D=1.01$ m)

$M_x = 26490 \text{ Nm} \quad M_y = 24686 \text{ Nm}$

Therefore  $\sigma = (8.66y + 8.07x)/B$

Then  $\sigma_7 = 4373/B \text{ N/mm}^2$

$\sigma_8 = 5279/B \text{ N/mm}^2$

$\sigma_9 = 5825/B \text{ N/mm}^2$

$\sigma_{10} = 5965/B \text{ N/mm}^2$

$\sigma_{11} = 5716/B \text{ N/mm}^2$

$\sigma_{12} = 5069/B \text{ N/mm}^2$

$\sigma_{13} = 4075/B \text{ N/mm}^2$

### Section BB (diameter $D=0.73$ m)

$M_x = 13347 \text{ Nm} \quad M_y = 11393 \text{ Nm}$

Therefore  $\sigma = (8.35y + 7.13x)/B$

Then  $\sigma_7 = 3048/B \text{ N/mm}^2$

$\sigma_8 = 3618/B \text{ N/mm}^2$

$\sigma_9 = 3941/B \text{ N/mm}^2$

$\sigma_{10} = 3989/B \text{ N/mm}^2$

$\sigma_{11} = 3834/B \text{ N/mm}^2$

$\sigma_{12} = 3303/B \text{ N/mm}^2$

$\sigma_{13} = 2602/B \text{ N/mm}^2$



From the above calculation it can be observed that the maximum tensile stress at each fuselage section occurs in stringer 10. Also the stress on stringer 10 (and all other stringers) is lower at section CC than at section AA and lower at section BB than at section CC. Therefore if fuselage frames are to be put at each of these sections much lighter stringers may be used between CC and BB than between used AA and CC. An additional frame will be positioned midway between AA and CC and between CC and BB to provide an extra strength and avoid buckling; the dimensions of frames between AA and CC will be designed differently from CC to BB. Also in the practical designing the frames will have to be slotted to allow the stringers to pass through. The arrangement is shown diagrammatically in Fig18.

1. The allowable direct stress in a stringer is  $124 \text{ N/mm}^2$ . The maximum direct stress is in stringer 10 at the section AA =  $7121/B \text{ N/mm}^2$ . Therefore required stringer area of cross-section is then given by

$$7121/B = 124$$

i.e.

$$B = 57.4 \text{ mm}^2$$

taking a approximate sectional area of  $60 \text{ mm}^2$  and designing the dimensions of the stringers as in Appendix A, the stringers with their dimensions is shown in fig 19

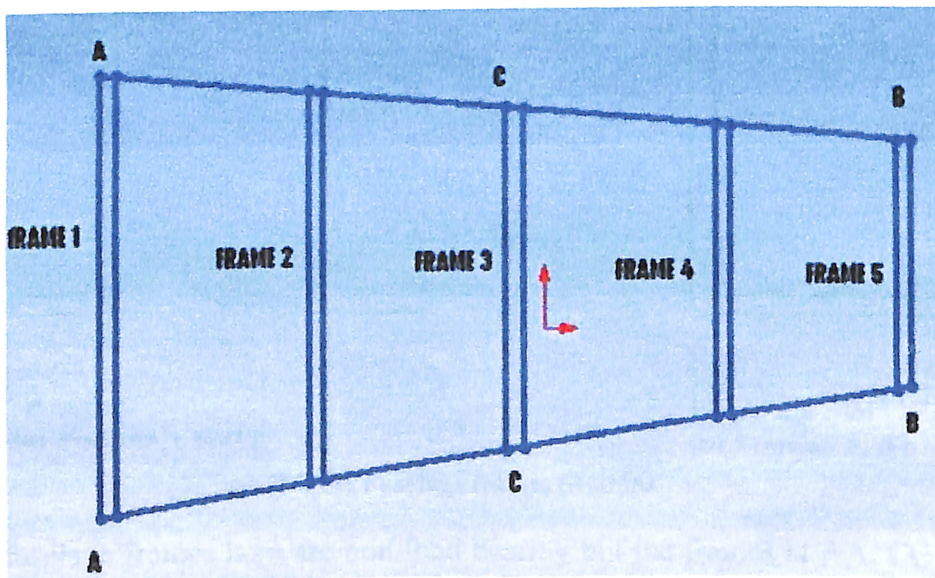


Fig18 :Placement of Frames in the fuselage

2. The maximum direct stress in stringer 10 at the section CC is  $5965/B \text{ N/mm}^2$ . The required stringer area is then given by:



$$5965/B = 124$$

i.e.  $B = 48.1 \text{ mm}^2$

The cross-section shown in Fig19. has a cross-sectional area of  $52 \text{ mm}^2$  and the design is made according to Appendix A and shown in Fig 20

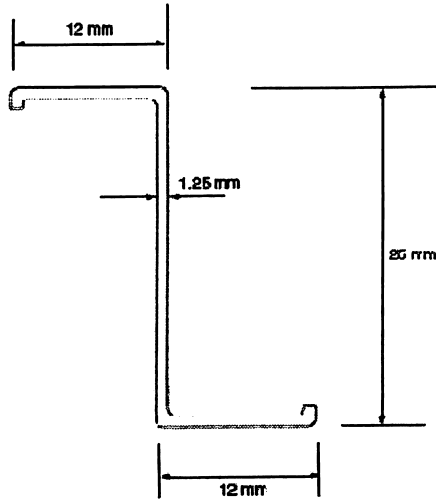


Fig19: Stringer section at AA to CC

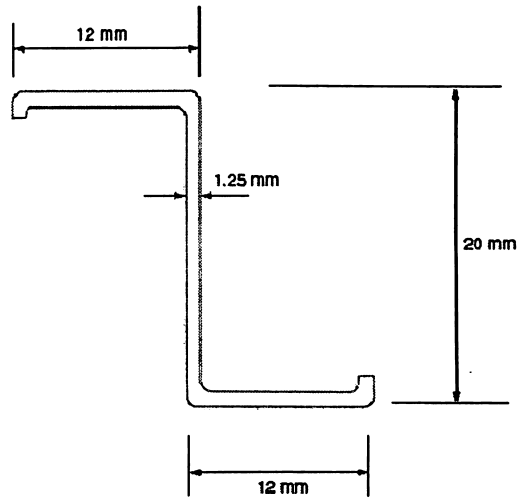
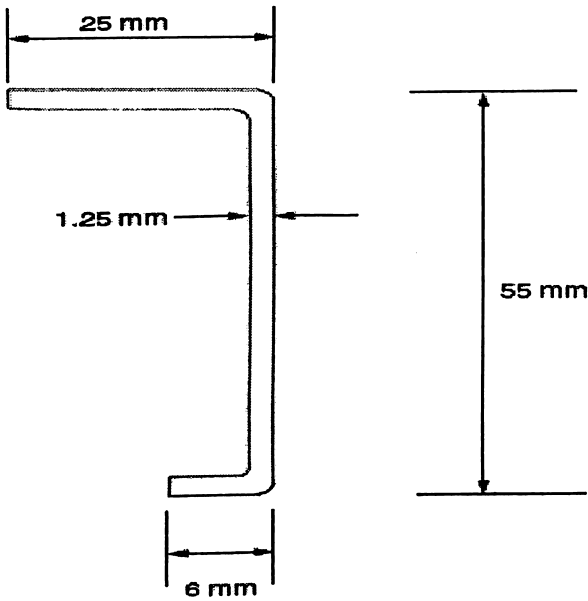
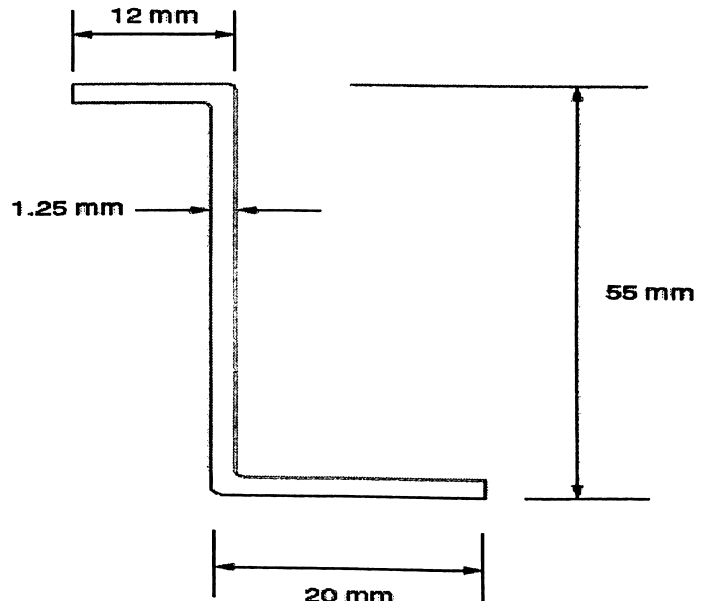


Fig 20: Stringer section at AA to CC



(a) Frames 1 and 5



(b) Frames 2, 3 and 4

Fig21: Fuselage frames Sections

Although the fuselage frames here are non-load bearing but the frames at AA, CC and BB must be of sufficient size to allow the ends of the stringers to be connected to them with help of brackets. Intermediate frames must also be of sufficient size to allow slots to be cut so that the stringers can pass through them. The frame sections to be used are shown in Fig 21.



## 2.5.2 Skin thickness

The fuselage cross-section is subjected to shear loads  $S_x$  and  $S_y$  along two perpendicular axes of symmetry. There is a change in shear flow as soon as crosssection differs. Equation that gives the change in shear flow as a boom, i.e. a stringer, is crossed is given and, due to symmetry, reduces to

$$q_2 - q_1 = -\left(\frac{S_x}{I_{yy}}\right)Bx - \left(\frac{S_y}{I_{xx}}\right)By$$

Then, since,  $B$  is constant for whole fuselage section and  $I_{xx} = I_{yy} = 3.0BD^2$  therefore above equation reduces to

$$q_2 - q_1 = -\left(\frac{S_x}{3.0D^2}\right)x - \left(\frac{S_y}{3.0D^2}\right)y$$

Considering the action of  $S_y$  (or  $S_x$ ) only. Then

$$q_2 - q_1 = -\left(\frac{S_y}{3.0D^2}\right)y$$

$$q_{23} = q_{12} + \left(\frac{S_y}{3.0D^2}\right) * 0.1294D = q_{12} + \frac{0.043S_y}{D}$$

$$q_{34} = q_{23} + \left(\frac{S_y}{3.0D^2}\right) * 0.25D = q_{12} + \frac{0.126S_y}{D}$$

$$q_{45} = q_{34} + \left(\frac{S_y}{3.0D^2}\right) * .353D = q_{12} + \frac{0.244S_y}{D}$$

$$q_{56} = q_{45} + \left(\frac{S_y}{3.0D^2}\right) * 0.433D = q_{12} + \frac{0.338S_y}{D}$$

$$q_{67} = q_{56} + \left(\frac{S_y}{3.0D^2}\right) * 0.483D = q_{12} + \frac{.549S_y}{D}$$

$$q_{76} = q_{67} + \left(\frac{S_y}{3.0D^2}\right) * 0.5D = q_{12} + \frac{0.716S_y}{D}$$

From symmetry  $q_{76} = -q_{67}$  so that

$$q_{12} + \frac{0.716S_y}{D} = -q_{12} - \frac{0.549S_y}{D}$$

$$q_{12} = -\frac{0.633S_y}{D}$$

Then

$$q_{23} = \frac{(-0.633 + 0.043)S_y}{D} = -\frac{0.59S_y}{D}$$

Also

$$q_{34} = -0.507S_y/D$$



$$q_{45} = -0.389S_y/D$$

$$q_{56} = -0.245S_y/D$$

$$q_{67} = -0.084S_y/D$$

Considering now the action of  $S_x$  only. Therefore

$$q_2 - q_1 = -\left(\frac{S_x}{3.0D^2}\right)x$$

$$q_{65} = q_{76} - \left(\frac{S_x}{3.0D^2}\right) * 0.1294D = q_{76} - 0.043 \frac{S_x}{D}$$

$$q_{54} = q_{65} - \left(\frac{S_x}{3.0D^2}\right) * 0.25D = q_{76} - 0.126 \frac{S_x}{D}$$

$$q_{43} = q_{54} - \left(\frac{S_x}{3.0D^2}\right) * 0.353D = q_{76} - 0.244 \frac{S_x}{D}$$

$$q_{32} = q_{43} - \left(\frac{S_x}{3.0D^2}\right) * 0.433D = q_{76} - 0.388 \frac{S_x}{D}$$

$$q_{21} = q_{32} - \left(\frac{S_x}{3.0D^2}\right) * 0.483D = q_{76} - 0.549 \frac{S_x}{D}$$

$$q_{12} = q_{21} - \left(\frac{S_x}{3.0D^2}\right) * 0.5D = q_{76} - 0.716 \frac{S_x}{D}$$

But  $q_{21} = -q_{12}$  from symmetry so that

$$q_{76} - 0.549S_x/D = -q_{76} + 0.716S_x/D$$

i.e.

$$q_{76} = 0.633S_x/D$$

Then

$$q_{65} = 0.59S_x/D$$

$$q_{54} = 0.507S_x/D$$

$$q_{43} = 0.389S_x/D$$

$$q_{32} = 0.245S_x/D$$

$$q_{21} = 0.084S_x/D$$

The shear flows due to  $S_x$  and  $S_y$  in skin panels 76 to 21 inclusive are in the same direction. An identical situation arises in panels 19 18 to 14 13 but in the remaining panels the shear flows are opposed. The shear flow produced by the applied torque is given by:

$$q = \frac{T}{2A}$$

Where, A is the area enclosed by the fuselage skin. Therefore



$$q = \frac{T}{2 \left( \frac{\pi D^2}{4} \right)}$$

or

$$q = \frac{0.637T}{D^2}$$

It can be seen from Table 3 that all the applied torques are positive, i.e. clockwise. The shear flow is then in the same sense in skin panels 76 to 21 as the shear flows due to  $S_x$  and  $S_y$ ; these panels are therefore subjected to the greatest shear stresses. The total shear flow in each of the panels 76 to 21 is then

$$q_{76} = 0.084S_y/D + 0.633S_x/D + 0.637T/D^2$$

$$q_{65} = 0.245S_y/D + 0.590S_x/D + 0.637T/D^2$$

$$q_{54} = 0.389S_y/D + 0.507S_x/D + 0.637T/D^2$$

$$q_{43} = 0.507S_y/D + 0.389S_x/D + 0.637T/D^2$$

$$q_{32} = 0.590S_y/D + 0.245S_x/D + 0.637T/D^2$$

$$q_{21} = 0.633S_y/D + 0.084S_x/D + 0.637T/D^2$$

From Table.3 the maximum values of  $S_y$ ,  $S_x$  and  $T$  at each section are produced by Case  $D_1$  in the flight envelope.

### Section AA (diameter $D=1.28$ m)

$$S_x = 12\,460 \text{ N}$$

$$S_y = 13\,350 \text{ N}$$

$$T = 34\,031 \text{ Nm}$$

Then,

$$q_{76} = 0.084 * \frac{13350}{1.28 * 10^3} + 0.633 * \frac{12460}{1.28 * 10^3} + 0.637 * 34031 * \frac{10^3}{(1.28 * (10^3))^2}$$

i.e.  $q_{76} = 20.3 \text{ N/mm}$

Similarly  $q_{65} = 21.5 \text{ N/mm}$

$$q_{54} = 22.2 \text{ N/mm}$$

$$q_{43} = 22.3 \text{ N/mm}$$

$$q_{32} = 21.8 \text{ N/mm}$$

$$q_{21} = 20.7 \text{ N/mm}$$

### Section CC (diameter $D=1.01$ m)

$$S_x = 12460 \text{ N}$$

$$S_y = 11837 \text{ N}$$

$$T = 34\,031 \text{ Nm}$$





Then,

$$q_{76} = 0.084 * \frac{11837}{1.01 \times 10^3} + 0.633 * \frac{12460}{1.01 * 10^3} + 0.637 * 34031 * \frac{10^3}{(1.01 * (10^3))^2}$$

i.e.  $q_{76} = 30.0 \text{ N/mm}$

Similarly  $q_{65} = 31.4 \text{ N/mm}$

$q_{54} = 33.1 \text{ N/mm}$

$q_{43} = 32.0 \text{ N/mm}$

$q_{32} = 31.2 \text{ N/mm}$

$q_{21} = 29.7 \text{ N/mm}$

### Section BB (diameter $D=0.73 \text{ m}$ )

$S_x = 12\,460 \text{ N}$

$S_y = 10547 \text{ N}$

$T = 34031 \text{ Nm}$

$$q_{76} = 0.084 * \frac{10547}{0.73 \times 10^3} + 0.633 * \frac{12460}{0.73 * 10^3} + 0.637 * 34031 * \frac{10^3}{(0.73 * (10^3))^2}$$

i.e.  $q_{76} = 52.7 \text{ N/mm}$

Similarly  $q_{65} = 54.3 \text{ N/mm}$

$q_{54} = 55.0 \text{ N/mm}$

$q_{43} = 54.6 \text{ N/mm}$

$q_{32} = 53.4 \text{ N/mm}$

$q_{21} = 51.3 \text{ N/mm}$

The skin will be of constant thickness so that the maximum shear stress in the skin will occur in the panel in which the shear flow is a maximum. This, from the above, is  $55.0 \text{ N/mm}$  in panel (54) at section BB. The maximum allowable shear stress is  $78 \text{ N/mm}^2$ , therefore

$$55.0 / t = 78$$

This gives

$$t = 0.71 \text{ mm}$$

A skin thickness of,  $0.75 \text{ mm}$  would not meet the requirements of a minimum thickness for rivet diameters equal to or greater than  $2.5 \text{ mm}$ . A skin thickness of  $1.0 \text{ mm}$  will therefore be used.



## Chapter 3

### CAD design and Procedure for analysis

**3.1 CAD Designing:** By the design procedure in last chapter we got the dimensions of frames, longerons, stiffeners and skin. Now the designing was done using CAD software solidworks. Since the fuselage is symmetric about both axis we can design only half of the fuselage and apply loads on it. Each component is modeled as a separate, three-dimensional part SolidWorks can calculate mass properties for each part, so the overall weight and inertial properties of the aircraft can be determined as the design progresses.

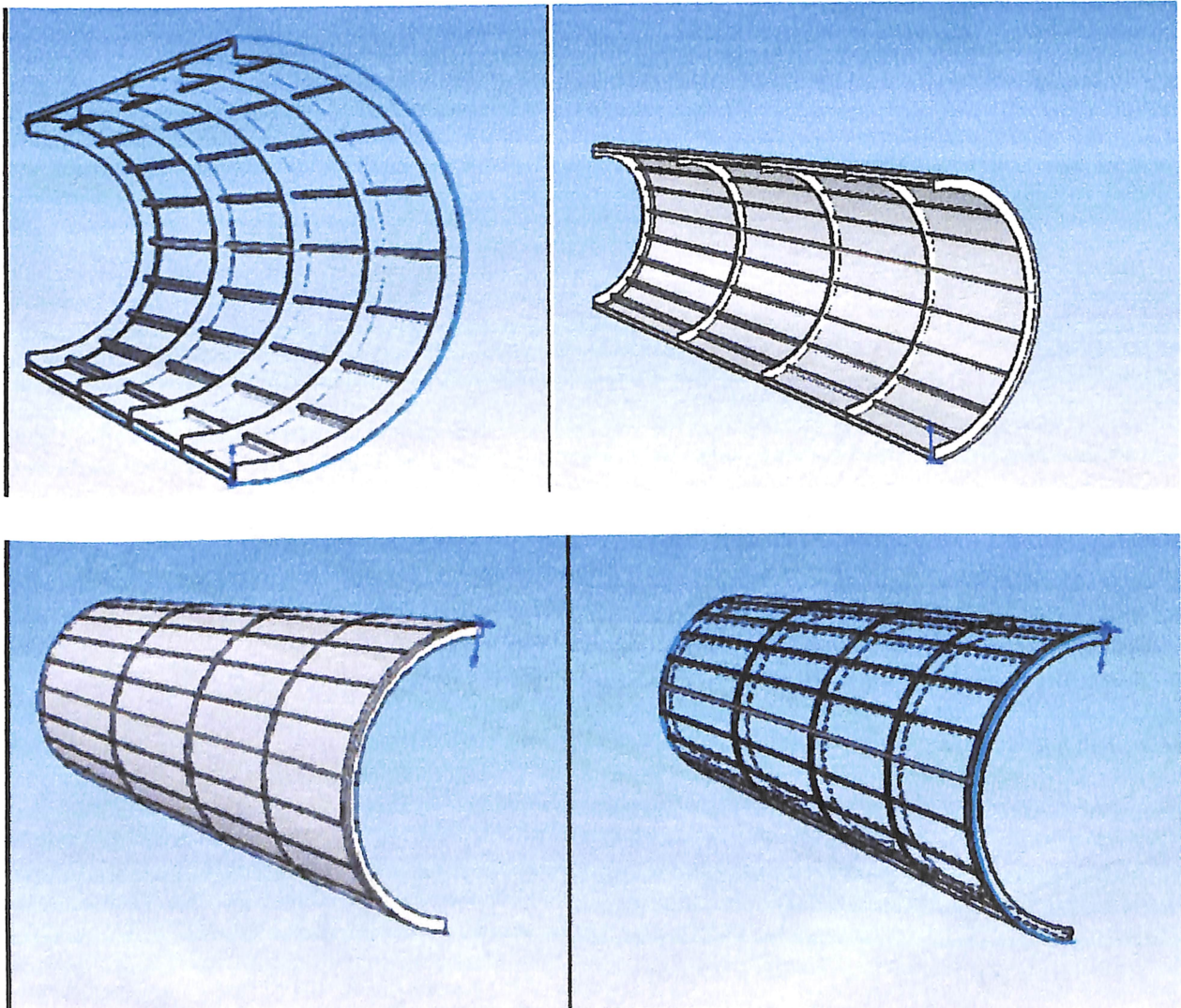
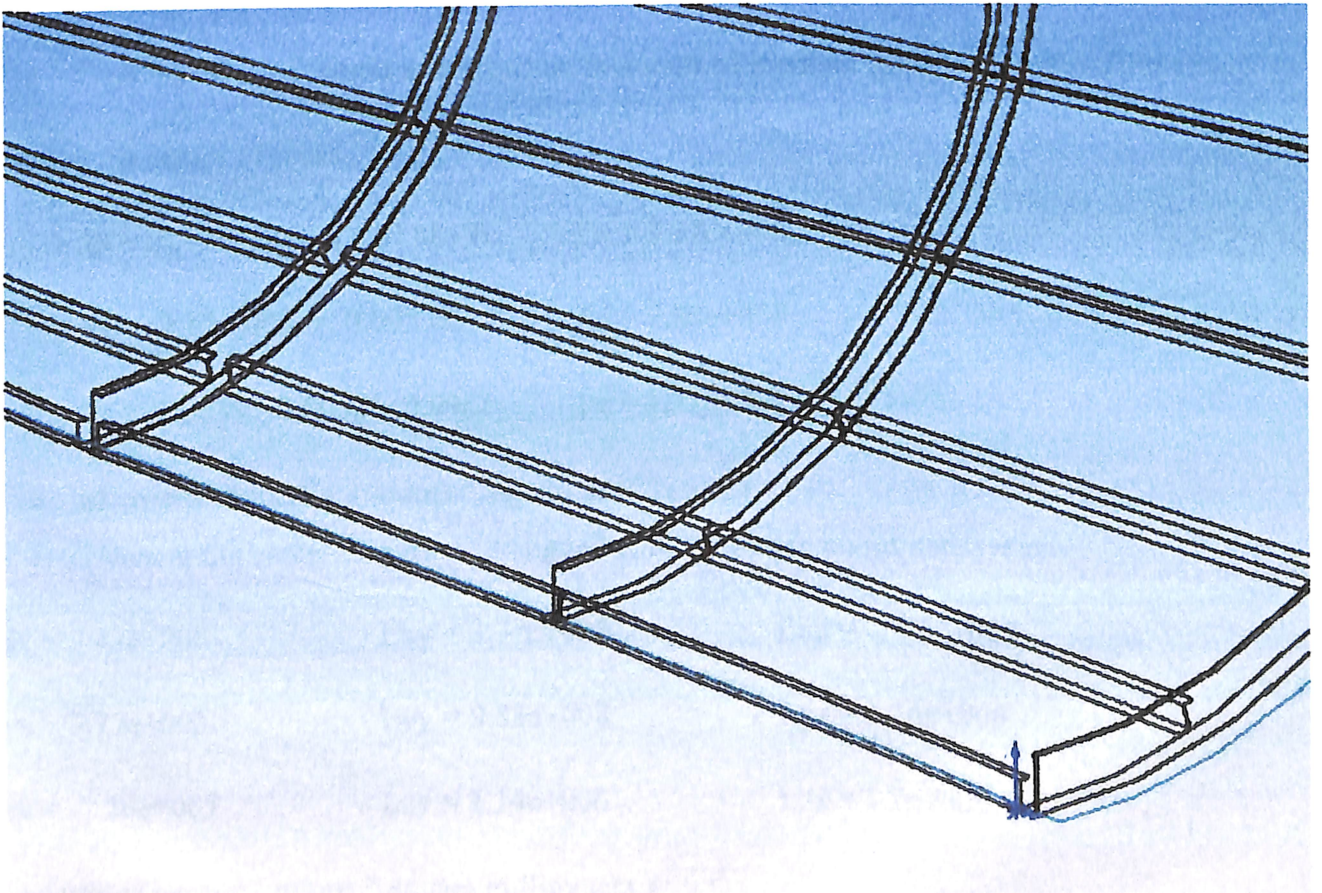
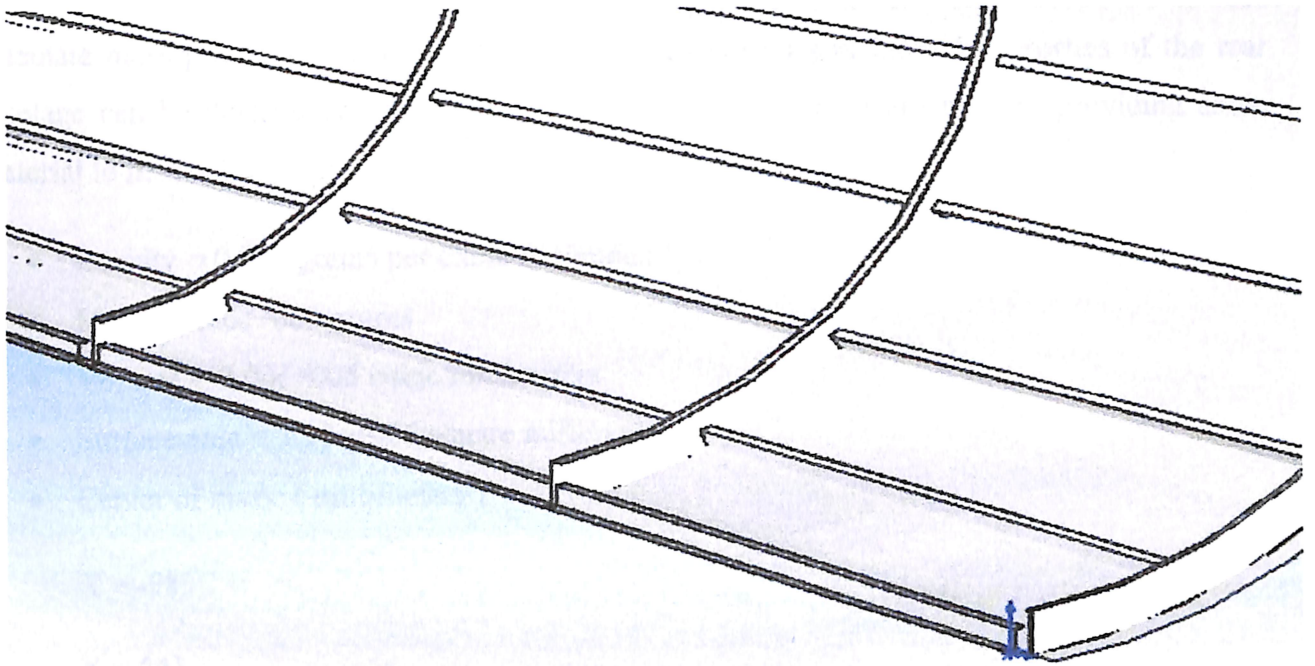


Fig 22: Wireframe and solid part view of the rear fuselage modeled in Solidworks



**Fig 23: Enlargement of a section showing the cross section of frames and stringers**



To create a solid model rear fuselage SolidWorks was used to take advantage of its advanced 3-D capabilities. The fuselage and individual components were modeled. SolidWorks can calculate mass properties for the part, so the overall weight and inertial properties of the rear fuselage can be determined. These are the part properties which are without providing some material to it.

- Density = 0.001 grams per cubic millimeter
- Mass = 2.06e+003 grams
- Volume = 2.06e+006 cubic millimeters
- Surface area = 3.38e+006 square millimeters
- Center of mass: ( millimeters )

$$X = -983$$

$$Y = 641$$

$$Z = -297$$

- Principal axes of inertia and principal moments of inertia: ( grams \* square millimeters )

Taken at the center of mass.

$$I_x = (0.996, 0.00043, -0.0839) \quad P_x = 3.37e+008$$

$$I_y = (2.13e-005, 1, 0.00538) \quad P_y = 9.88e+008$$

$$I_z = (0.0839, -0.00536, 0.996) \quad P_z = 1.21e+009$$

- Moments of inertia: ( grams \* square millimeters )

1. Taken at the center of mass and aligned with the output coordinate system.

$$L_{xx} = 3.43e+008 \quad L_{xy} = 3.77e+005 \quad L_{xz} = -7.26e+007$$

$$L_{yx} = 3.77e+005 \quad L_{yy} = 9.88e+008 \quad L_{yz} = 1.14e+006$$

$$L_{zx} = -7.26e+007 \quad L_{zy} = 1.14e+006 \quad L_{zz} = 1.2e+009$$

Moments of inertia: ( grams \* square millimeters )



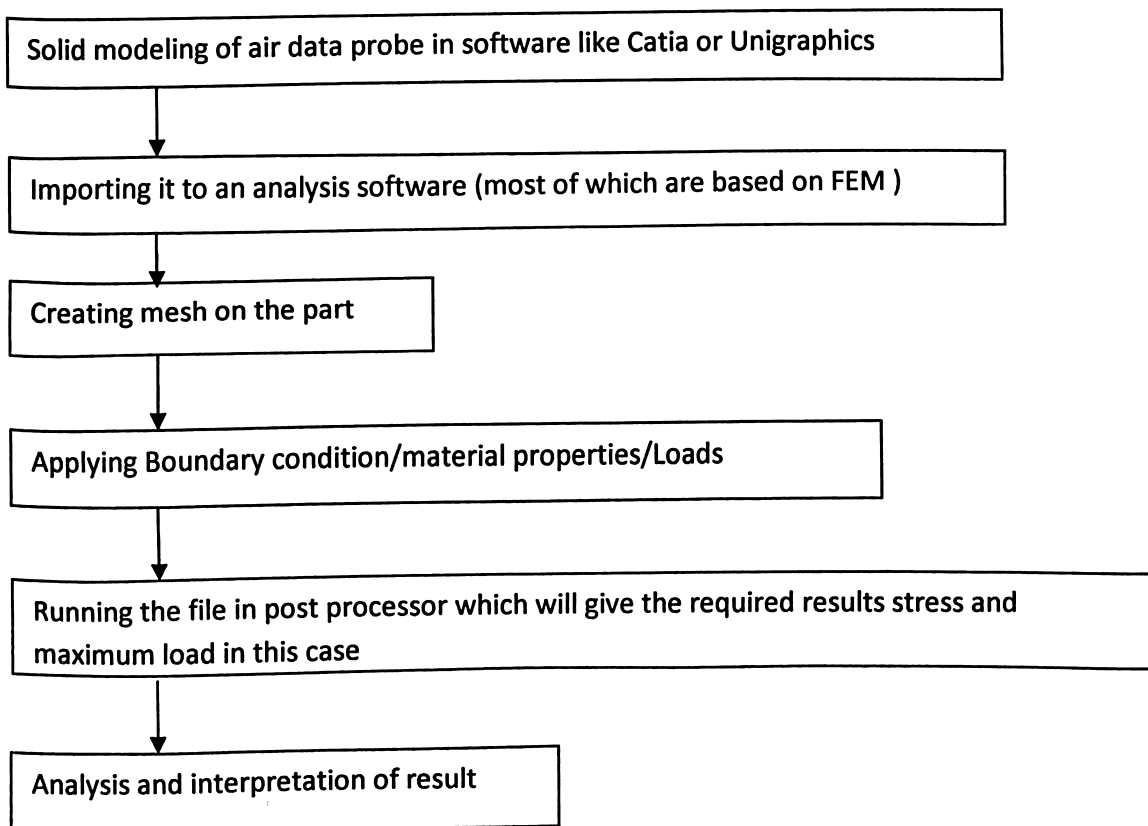
## 3.2 Analysis

### 3.2.1 Analysis Objective

- Designing of structural components with major criterion of strength requirements
- Cross Checking a given design solution for strength / stiffness with the software results
- Design refinement , making a more durable design
- Shape and material design optimization for better functionality, strength, weight, and manufacturing cost, requirements and limitations

### 3.2.2 Analysis Procedure

The procedures to be followed for stress analysis of rear fuselage using FEM will be:



Since the analysis of the Fuselage is done on Ansys which is based on Finite element method (FEM), to understand the procedures to be followed the first requisite is to understand the principals and working of finite element analysis.



### 3.3 Finite element method

**3.3.1 Principle:** A body has infinite points but it is very difficult to analyze the structure for each point, so the whole part is divided into small elements, finite in number which can deform independently and can have different material and geometrical properties. Hence the method is called “finite element” method.

Finite element formulation of the problem results in a system of simultaneous algebraic equation for solution rather than requiring the solution for differential equation. This approximate method gives approximate values of the unknown at discrete number of points in the continuum. It is a numerical method of finding out the solution to any engineering problem (problems refers to solving differential equations, integral equations etc.).

This is the process of modeling a body by dividing it into number of smaller bodies or units interconnected at a point common to two or more elements (nodal points or nodes), or boundary line or surface, this process is called discretization. The equation is formulated for each element and is combined to obtain a solution for whole body. Displacement at each node is determined and the stress within each element is noted. In nonstructural problems nodal unknown may be temperature or fluid pressure.

The formation of elements on the part is called meshing elements can be 1D, 2D or 3D depending upon body to be meshed.

1D element include beam element & rod element, 2D elements include triangle or quadrilateral element having 3 and 4 nodes respectively, each element type has its advantages and disadvantages like area covered, stiffness of the element, etc. 3D elements are used for a solid body they include tetrahedral (4 nodes), brick element (8 nodes) and wedge element (6 nodes).

#### 3.3.2 Working procedure of Finite element method:-

1. Discretization (or representation) of the given domain into a collection of preselected finite elements. (This step can be postponed until the finite element formulation of the equation is completed.)
  - (a) Construct the finite element mesh of preselected elements.



- (b) Number the nodes and the elements.
- (c) Generate the geometric properties (e.g., coordinates and cross-sectional areas) needed for the problem.
2. Derivation of the element equations for all typical elements in the mesh.
  - (a) Construct the variational formulation of the given differential equation over the typical element.
  - (b) Assume that a typical dependent variable  $u$  is of the form  $U = \sum u_i \Psi_i$  and substitute it into step 2a to obtain element equations in the form  $\{K^e\} \{u^e\} = \{F^e\}$
  - (c) Select, if already available in the literature, or derive element interpolation functions  $\Psi_i$  and compute the element matrices.
3. Assembly of element equations to obtain the equations of the problem.
  - (a) Identify the inter element continuity conditions among the primary variables (relationship b/w the local degree of freedom and the global degree of freedom-connectivity of the elements) by relating nodes to global nodes.
  - (b) Identify the “equilibrium” conditions among the secondary variables (relationship b/w the local source/ force components and the globally specified source comp.)
4. Imposition of the boundary conditions of the problems.
  - (a) Identify the specified global primary degrees of freedom.
  - (b) Identify the specified global secondary degrees of freedom
5. Solution of the assembled equations.
6. Post processing of the results.
  - (a) Compute the gradient of the solution or the other desired quantities from the primary degrees of freedom computed in step 5.
  - (b) Represent the results in tabular and/or graphical form.



### 3.3.3 Advantages of finite element analysis over traditional methods:

- Disciplines such as aeronautical, biomechanical, and automotive industries commonly use integrated FEM in design and development of their products. In a structural simulation, FEM helps tremendously in producing stiffness and strength visualizations and also in minimizing weight, materials, and costs.
- FEM allows detailed visualization of where structures bend or twist, and indicates the distribution of stresses and displacements.
- FEM software provides a wide range of simulation options for controlling the complexity of both modeling and analysis of a system.
- FEM allows entire designs to be constructed, refined, and optimized before the design is manufactured.
- This powerful design tool has significantly improved both the standard of engineering designs and the methodology of the design process in many industrial applications.
- The introduction of FEM has substantially decreased the time to take products from concept to the production line.
- It is primarily through improved initial prototype designs using FEM that testing and development have been accelerated.
- FEM has resulted in enhanced accuracy, design and better insight into critical design parameters, virtual prototyping, and fewer hardware prototypes, resulting in a faster and less expensive design cycle which has increased productivity and has increased revenue.

## 3.4 CAD to Ansys Solving

**3.4.1 Geometry:** We used Ansys for finite element analysis (FEA). It includes a fully integrated the designing, meshing and solver for developing finite element models and reviewing the results. Although Ansys can directly read geometry created by SolidWorks, we can first translate the geometry into a neutral format, and then import it into the modeler. Solid models are often too detailed for finite element analysis, and this approach allowed us to more easily restrict the model to





the portions we needed. Both STEP and IGES files (neutral formats developed by the CAD industry for data exchange between programs) were used for the conversion.

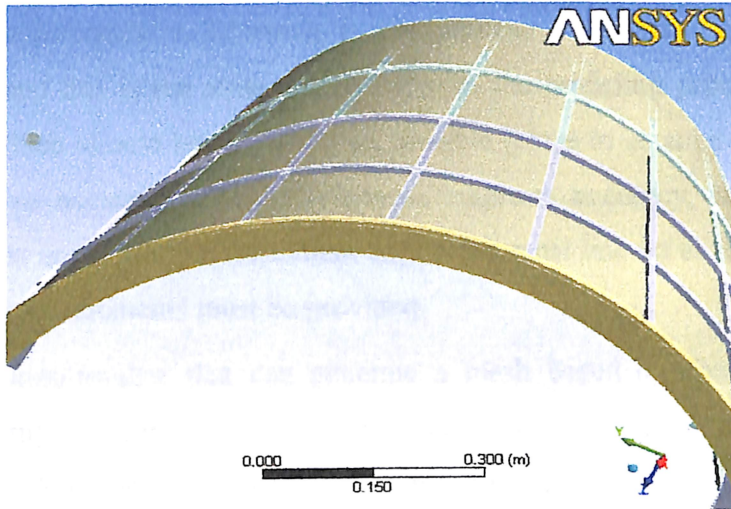


Fig 24: Part imported in Ansys

### 3.4.1 Material properties

Aluminum alloy is given as the material to the part having properties as

Data Overview

Aluminum Alloy 2

**Structural** Add/Remove Properties

<input type="checkbox"/> Young's Modulus	7.1e+010 Pa	
<input type="checkbox"/> Poisson's Ratio	0.33	
<input type="checkbox"/> Density	2770. kg/m <sup>3</sup>	
<input type="checkbox"/> Thermal Expansion	2.3e-005 1/°C	
<input type="checkbox"/> Alternating Stress		
<input type="checkbox"/> Tensile Yield Strength	2.8e+008 Pa	
<input type="checkbox"/> Compressive Yield Strength	2.8e+008 Pa	
<input type="checkbox"/> Tensile Ultimate Strength	3.1e+008 Pa	
<input type="checkbox"/> Compressive Ultimate Strength	0. Pa	

**Thermal** Add/Remove Properties

<input type="checkbox"/> Thermal Conductivity		
<input type="checkbox"/> Specific Heat	875. J/kg·°C	

**Electromagnetics** Add/Remove Properties

<input type="checkbox"/> Relative Permeability	1.	
<input type="checkbox"/> Resistivity	5.7e-008 Ohm·m	

**Thermal Conductivity**

**Alternating Stress**

Fig 25 Material properties of Aluminum alloy used in the rear fuselage



### 3.4.3 Mesh Generation

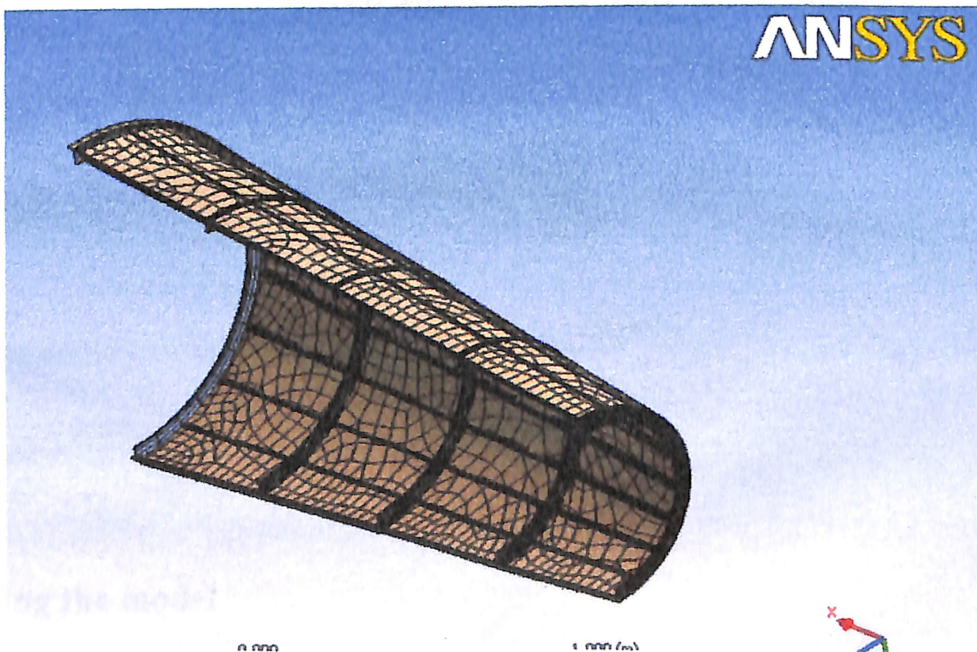
Finite element modeling works by dividing a complex surface into smaller, simpler elements that can be modeled mathematically. Meshing is the process of overlaying elements onto the geometric surfaces and this is the most difficult step in the modeling process. Element shapes should be defined which should be as regular as possible (close to squares or triangles for shell elements) to improve accuracy. Smaller elements improve accuracy, but solution time or memory requirements are much more. Element edges also must line up at geometric boundaries. For solid geometries 3 D elements must be provided

Ansys includes an auto-mesher that can generate a mesh based on general parameters like element size or density. It even has features like size grading that places small elements itself where accuracy is needed. For our model statistics of mesh are:

**No.of Nodes= 72307**

**No. of Elements: 35719**

The most common measure of model size is the total degrees of freedom, it is the number of possible translational and rotational motions possible at the model nodes. It is estimated that degree of freedom is about 6 to 12 times the number of elements.



**Fig 26 Meshed geometry**



### 3.4.2 Boundary conditions

Geometric boundary conditions were given to the model after meshing. Boundary conditions fix the model in space to prevent rigid body translations or rotations from affecting the analysis, and to represent how different parts attach to each other. The fuselage was made fixes at wing and Tail attachments. Also stringer and Frames were fixed to skin. As we have made half fuselage the edge should be given symmetric conditions

**3.4.3 Loads:** Loads can be applied to the nodes for the mesh or the geometry we have applied loads to the nodes. These loads are according to the calculate loads, torque and moment in the preceding chapter. The torque and moments are replaced y corresponding forces.

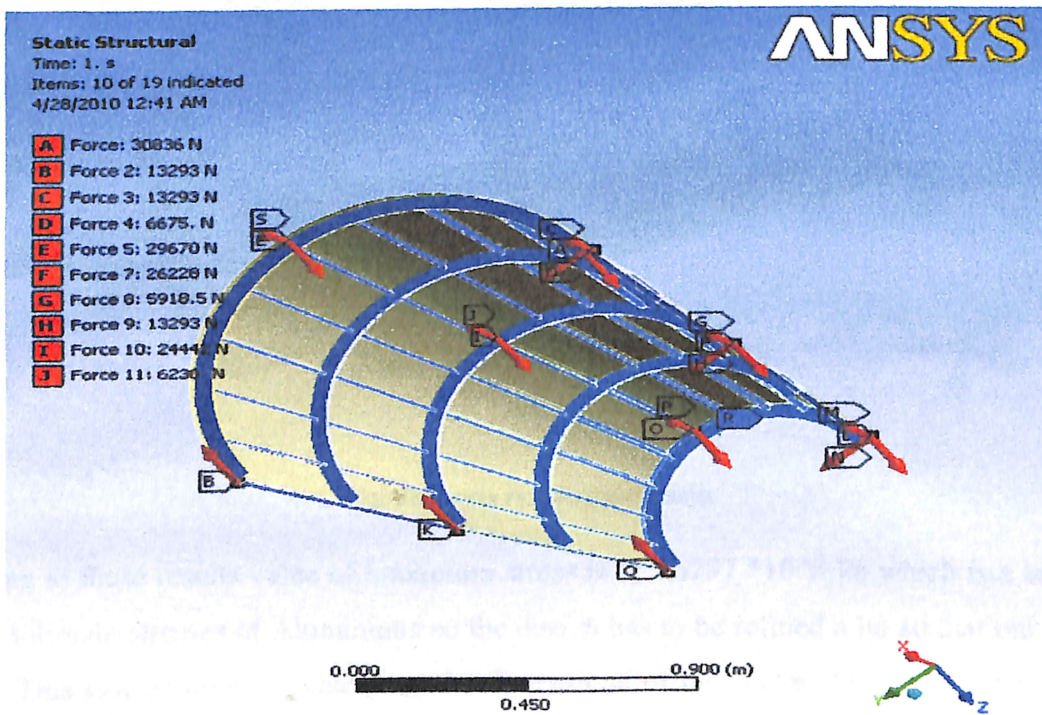


Fig 27 Applied load on the geometry

### 3.4.5 Running the model

The model was then solved using Ansys solver and Results for the Stress were achieved.



## Chapter 4

### Results

**4.1 Static analysis** was done on the part and the results for distribution of stress were found as:

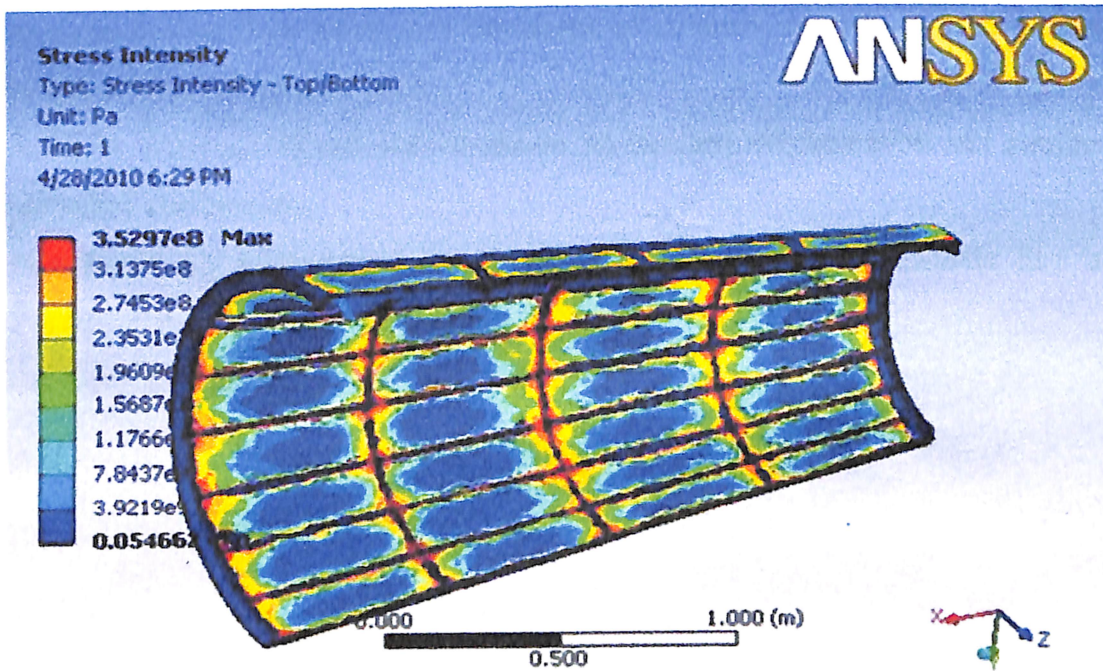


Fig: 28 Stress results from ansys

According to these results value of maximum stress is at  $3.5297 \times 10^8$  Pa which is a bit greater than the ultimate stresses of Aluminums so the design has to be refined a bit so that our part will not fail. This can be done by changing the dimensions of the frames taken and designing and again. Though results show that the maximum stresses are located at very few places and areas, especially at those places where we have applied loads.

### 4.2 Conclusion

The designing is very close to the safe designing but a few changes in design of frames and stingers have to be made so that it could withstand this much amount of stress.



## Chapter 5

### Scope of Work

- The design can be altered and new calculations and analysis can be done by new stringers and frame design.
- The analysis can be done using composite material.
- Analysis and design of whole fuselage can be done.
- Analysis can be done using different shape of stringers and Frame to choose what the best combination could be used
- A whole different aircraft can be chosen as baseline configuration and similar design procedure can be done.
- Design can be done considering Impact loads and other phenomena like buckling dynamic effects etc.
- Different tail plane and wing configuration can be considered.



# Appendix 'A'

## Section properties of Typical Aircraft Extruded section

Sect. No.	Dimensions							Area in. <sup>2</sup>	Properties about X-X			Properties about Y-Y			
	A	B	C	t	t <sub>1</sub>	t <sub>2</sub>	R		I <sub>xx</sub>	C <sub>xx</sub>	e	I <sub>yy</sub>	C <sub>yy</sub>	e'	
	1	17/32	13/16	1/8	.040	.050	1/16	.088	.00888	.317	.319	.00150	.148	.112	
	2	1/2	1	7/32	.030	.050	.050	.113	.01515	.3665	.610	.00159	.117	.110	
	3	1/2	7/8	3/16	1/16	1/16	3/32	.1029	.0100	.313	.383	.00162	.125	.116	
	4	9/16	7/8	3/16	.060	.060	3/32	.1031	.0102	.313	.383	.00216	.114	.130	
	5	9/16	7/8	3/16	.050	.078	3/32	.1061	.0120	.329	.346	.00302	.165	.133	
	6	3/4	1-1/8	3/16	.075	.075	1/16	.1514	.0239	.397	.455	.00550	.190	.182	
	7	3/4	1-5/16	9/32	3/32	3/32	9/64	.230	.0507	.471	.574	.0086	.187	.171	
	8	7/8	3/8		1/16	1/16	1/16	.095	.0101	.328	.4375	.00111	.108	.108	
	9	7/8	3/8		3/32	3/32	3/32	.1375	.0136	.314	.4375	.00148	.104	.118	
	10	1	1/2		3/32	3/32	3/32	.1736	.0240	.371	.500	.00376	.147	.158	
	11	1-1/4	11/16		3/32	3/32	3/32	.2320	.0536	.480	.625	.00992	.206	.210	
	12	1-13/16	1		3/16	3/16	3/32	.644	.1920	.677	.905	.0555	.289	.326	
	13	2-1/2	1		1/8	1/8	1/8	.581	.525	.950	1.25	.0462	.338	.323	
	14	3-1/4	1-1/8		1/8	3/16	1/8	.666	.524	.888	1.125	.0872	.352	.375	
	15	1-3/8	11/16	1/8	1/16	1/16	1/16	.137	.00425	.178	.133	.0178	.360	.687	
	16	1-5/8	3/4	5/32	1/16	1/16	1/16	.168	.00580	.166	.133	.0339	.446	.812	
	17	1-5/8	3/4	5/32	1/16	3/32	5/64	.206	.00689	.170	.131	.0458	.474	.812	
	18	2	1-1/16	3/16	3/32	3/32	3/32	.306	.0252	.267	.201	.0781	.509	1.0	
	19	2	1-1/4	3/16	1/8	3/32	3/32	.359	.0475	.264	.200	.0827	.479	1.0	
	20	2	1-5/32	1/4	3/32	1/8	1/8	.378	.0291	.279	.217	.1095	.541	1.0	
	21	2-5/16	1-1/2	5/16	5/32	5/32	5/32	.668	.1075	.401	.336	.2326	.590	1.166	
	22	1-1/2	1-1/8		1/16	1/16	1/16	.161	.0186	.342	.255	.0171	.326	.789	
	23	1-1/2	1		1/8	3/32	1/8	.384	.1562	.641	.658	.0346	.302	.750	
	24	1-3/4	1		9/32	1/8	1/8	.760	.315	.643	.757	.0599	.381	.878	
	25	1-3/4	2-9/16		1/4	5/32	3/32	.818	.660	.809	.959	.0720	.282	.675	
	26	2	2-1/2		5/32	1/8	5/32	.605	.365	.800	.793	.159	.314	1.00	
	27	11/16	1	1/8	.030	.030	.030	3/32	.135	.0218	.404	.300	.0183	.348	.6625
	28	13/16	1-1/8	3/16	.060	.060	.050	3/32	.180	.0375	.456	.362	.0295	.405	.7825
	29	3/8	1-1/2	3/16	.030	1/16	1/16	3/32	.166	.0603	.395	.75	.0151	.298	.600
	30	3/4	2	1/4	.080	1/16	1/16	3/32	.214	.1282	.792	1.00	.0281	.356	.725
	31	3/4	2	1/4	1/16	1/16	1/16	3/32	.2378	.1449	.772	1.00	.0282	.361	.719
	32	3/4	2	1/4	1/16	3/32	3/32	3/32	.2655	.1829	.795	1.00	.0385	.356	.719
	33	9/16	7/8		.050	.050	1/16	.089	.0120	.349	.437	.0056	.228	.537	
	34	1/2	7/8		1/16	1/16	1/16	.115	.0135	.341	.437	.00446	.197	.469	
	35	9/16	1		1/16	.080	3/32	.151	.0239	.397	.500	.00828	.234	.531	
	36	9/16	1 1/4		.075	.075	3/32	.170	.0384	.475	.625	.00876	.227	.525	
	37	11/16	1-1/4		1/16	.100	3/32	.213	.0550	.508	.625	.0194	.302	.655	
	38	.825	1-1/2		.075	.120	3/32	.302	.116	.604	.750	.0391	.360	.787	
	39	3/4	2		.072	.072	1/8	.249	.1446	.757	1.00	.0180	.387	.714	
	40	3/4	2		.102	.102	3/32	.330	.1826	.750	1.00	.0238	.385	.700	



# Appendix 'B'

## Analysis and Design of component

TABLE 1 - Continued		Elliptical Ring
<p>Area = <math>t(d + 2a)</math>  <math>\bar{y} = d/2</math>  <math>I_x = \frac{bd^3}{12} - \frac{a(d - 2ct)^3}{12}</math>  <math>I_y = d(b + a)^2 - 2a^2c - 6ab^2c</math>  <math>\tan 2\theta = \frac{(dt - ct^2)(b^2 - bt)}{I_x - I_y}</math>            For <math>I_{x-x}</math> and <math>I_{y-y}</math>, see equations as given for angle.</p>	<p>Area = <math>\pi(a_1b_1 - a_2b_2)</math>  <math>I_x = \frac{\pi}{4}(a_1^3b_1 - a_2^3b_2)</math>  <math>e_x = \sqrt{\frac{I}{Area}}</math></p>	
<p>Area <math>A = 2ct_1 + at</math>  <math>\bar{y} = \frac{d^2t_1 + .5t^2a}{A}</math>  <math>I_x = \frac{2ct_1d^3 + at^3}{3} - A\bar{y}^2</math></p>	<p>Area = <math>.215 a^2</math>  <math>\bar{x} = .223 a</math></p>	
<p>Circle            Area = <math>\pi r^2</math>  <math>I_{x-x} = \frac{\pi r^4}{4}</math>  <math>I_{y-y} = \frac{5\pi r^4}{4}</math>  <math>e_{x-x} = \frac{r}{2}</math></p>	<p>Parabolic Fillet            Area = <math>\frac{x_1 y_1}{3}</math>  <math>\bar{x} = \frac{3}{4} x_1</math>  <math>\bar{y} = .3 y_1</math></p>	
<p>Ring            Area = <math>\pi(r_2^2 - r_1^2)</math>  <math>I_{x-x} = \frac{\pi}{4}(r_2^4 - r_1^4)</math>  <math>e_{x-x} = \sqrt{\frac{r_2^4 + r_1^4}{2}}</math></p>	<p>Parabolic            Area = <math>x_1 y_1 / 4</math>  <math>\bar{x} = \frac{4}{3} x_1</math>  <math>\bar{y} = \frac{2}{3} y_1</math></p>	
<p>Semi-circle            Area = <math>\frac{\pi r^2}{2}</math>  <math>\bar{y} = .424 r</math>  <math>I_x = .1098 r^4</math>  <math>e_x = .284 r</math></p>	<p>TABLE 2 Properties of Solids</p>	
<p>Semi-circular Ring            Area = <math>\frac{\pi}{2}(R^2 - r^2)</math>  <math>y = \frac{4}{3\pi} \frac{(R^2 + Rr + r^2)}{R + r}</math>, approx. <math>\bar{y} = \frac{2r}{\pi}</math>  <math>I_x = .1098(R^2 - r^2) - \frac{.283Rr^2}{R + r} - \frac{.283r^3}{R + r}</math>  <math>I_x(\text{approx}) = \frac{.3t(r + R)^2}{2}</math> when <math>\frac{t}{R}</math> is small</p>	<p>Solid Circ. Cyl.            Vol. = <math>\pi r^2 L</math> (r = radius)  <math>M = \rho V</math> (Total wt.)  <math>I_{x-x} = \frac{Mr^2}{2}</math>  <math>I_{y-y} = M[r^2 + (L/3)^2]^{1/2}</math></p>	
<p>Ellipse            Area = <math>\pi ab</math>  <math>I_x = \frac{\pi a^3 b}{4}</math>  <math>e_x = \frac{a}{2}</math></p>	<p>Hollow Circ. Cyl.            Vol. = <math>\pi L(r_2^2 - r_1^2)</math>  <math>I_{x-x} = M(r_1^2 + r_2^2)/2</math>  <math>I_{y-y} = M(r_1^2 + r_2^2 + L^2/3)/4</math>            For thin hollow circ. cyl.  <math>I_{x-x} = Mr^2</math></p>	
	<p>Rect. Prism            Vol. = <math>abL</math>, <math>M = \frac{W}{g}</math>  <math>I_{x-x} = M(a^2 + b^2)/12</math>  <math>I_{y-y} = ML^2/12</math>  <math>I_{z-z} = ML^2/3</math></p>	
	<p>Sphere  <math>r_1 = \text{radius of sphere}</math>            Solid Sphere Vol. = <math>(4\pi r^3)/3</math>  <math>I_{\text{about axis}} = \frac{21}{5} r^2</math>            Thin Hollow Sphere Vol. = <math>\frac{4\pi}{3}(r_1^3 - r_2^3)</math>  <math>I_{\text{about dia.}} = \frac{21}{3} r^2</math></p>	



## References

- [1] Bauchau, O.A., and Craig, J.I., "Structural Analysis with Applications to Aerospace Structures".
- [2] Patnaik, Surya N., and Hopkins, Dale A., "Strength of Materials".
- [3] Megson, T. H. G., "Aircraft Structures for engineering students", Fourth Edition, pp. 767-796.
- [4] Bruhn, Elmer, "Analysis and Design of missile Structures/Airplane Design", pp, A3-3 and A3-14.
- [5] Barnard, R.W., and Kellog, C., "Application of Convolution Operators to Problem in Univalent Theory", Michigan.
- [6] Mach, J., "Carbon Fibre Composite Airplane Fuselage : Concept and Analysis".
- [7] Boris, Ya., Koleniskov, Lars Herbeck German aerospace Center(DLR) Germany.
- [8] AIAA- 2001-1326 "Skin, Stringer and Fastener Loads in Buckled Fuselage Panels".
- [9] Young, R.D., Rose, C.A., And Starnes. J.H. Jr., NASA Langley Research center. 42<sup>nd</sup> AIAA /ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference April 16-2001
- [9] Gerard, George, "Minimum Weight Analysis of Compression Structures"1956 ch.2,3 and 7.
- [10] Gordon, J.E., "Structures of Why Things Don't Fall Down", 1978, Ch. 13-14.
- [11] Shanley, F.R., "Weight Analysis Of Aircraft Structures", 1960, Ch. 1-4.
- [12] Hutton, David V., "Fundamentals of FEM", Fourth edition.