Name: **Enrolment No:**	UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2020

Course: Group Theory I | **Semester: III**
Programme: B.Sc. (Hons.) Mathematics | **Time: 03 hrs.**
Course Code: MATH 2028 | **Max. Marks: 100**

SECTION A

Instructions: Attempt all questions. Each question will carry 5 marks.

S. No.	Question	CO
Q1	If $f = (1\ 2\ 3)$, $g = (2\ 4\ 3)$ and $h = (1\ 3\ 4)$ are three permutations on $1, 2, 3, 4, 5, 6$; then the product fgh is equal to A. $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 5 \\ 1 & 2 & 3 & 4 & 6 & 6 \end{pmatrix}$ B. $\begin{pmatrix} 1 & 2 & 3 & 4 & 6 & 5 \\ 5 & 6 & 2 & 1 & 5 & 6 \end{pmatrix}$ C. $\begin{pmatrix} 1 & 2 & 5 & 1 & 4 & 3 \\ 1 & 6 & 5 & 1 & 3 & 5 \end{pmatrix}$ D. $\begin{pmatrix} 1 & 2 & 5 & 3 & 6 & 4 \\ 1 & 2 & 3 & 4 & 5 & 6 \end{pmatrix}$	**CO1**
Q2	If the elements a, b of a group commute and $O(a) = m$, $O(b) = n$, where m and n are relatively prime, then $O(ab)$ is A. m B. n C. mn D. $\frac{m}{n}$	**CO2**
Q3	Which one is False? A. Every quotient group of a cyclic group is cyclic and the converse is not true. B. Every quotient group of a commutative group is abelian and the converse is also true. C. If Z denote the centre of a group G and G/Z is cyclic then G is abelian. D. None of the above	**CO3**
Q4	How many generators are there of the cyclic group G of order 8? A. 1 B. 2 C. 3 D. 4	**CO3**

Q5	If G is the additive group of integers and H is the subgroup of G obtained on multiplying the elements of G by 5, then the index of H in G is A. 2 B. 3 C. 5 D. 7	**CO4**
Q6	The mapping $f: C \rightarrow R$ such that $f(x+iy) = x$ is a homomorphism of the additive group of complex numbers onto the additive group of real numbers. The kernel of f consists of all complex numbers whose A. Real part is zero B. Imaginary part is zero C. Modulus is one D. None of the above	**CO5**
SECTION B **Instructions: Attempt all questions. Each question will carry 10 marks. Question 11 has internal choice.**		
Q7	Show that the set of six transformations $f_1, f_2, f_3, f_4, f_5, f_6$ on the set of complex numbers defined by $f_1(z) = z, f_2(z) = \frac{1}{z}, f_3(z) = 1 - z, f_4(z) = \frac{z}{z-1}, f_5(z) = \frac{1}{1-z}, f_6(z) = \frac{z-1}{z},$ forms a finite non-abelian group of order six with respect to the composition known as composite of two functions or product of two functions.	**CO1**
Q8	If in a group G, $xy^2 = y^3x$ and $yx^2 = x^3y$, then show that $x = y = e$ where e is the identity of G.	**CO2**
Q9	Prove that, a subgroup H of a group G is a normal subgroup of G if and only if each left coset of H in G is a right coset of H in G. Also, show that every subgroup of an abelian group is normal.	**CO3**
Q10	State and prove Lagrange's theorem. Use Lagrange's theorem to prove that a finite group cannot be expressed as the union of two of its proper subgroups.	**CO4**
Q11	If p is a prime number and G is a non abelian group of order p^3, show that the centre of G has exactly p elements. **OR** Prove that, every group of prime order is cyclic.	**CO4**
SECTION C **Instructions: Attempt all questions. Each question will carry 20 marks. Question 12 has internal choice.**		
Q12	Define Homomorphism of groups and Kernel of a Homomorphism. Prove that every homomorphic image of a group G is isomorphic to some quotient group of G. Also, show that every homomorphic image of an abelian group is abelian and converse is not true. **OR** State and prove, Second and Third law of Isomorphism.	**CO5**