Name:

Enrolment No:

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2020

Course: Theory of Real Functions | **Semester: III**

Program: B.Sc. (Hons.) Mathematics | **Time : 03 hrs.**

Course Code: MATH 2010 | **Max. Marks: 100**

Instructions: All questions are compulsory.

SECTION A

1. **Each question carries 5 marks.**
2. **Complete the statement / Select the correct answers(s).**

S. No.		CO
Q1	Let $f: \mathbb{R} \to \mathbb{R}$ be a differentiable function such that $f(2) = 2$ and $\|f(x) - f(y)\| \leq 5(\|x - y\|)^{3/2}$ for all $x, y \in \mathbb{R}$. Let $g(x) = x^3 f(x)$. Then $g'(2) =$________	**CO1**
Q2	Let $f: \mathbb{R} \to [0, \infty)$ be a continuous function. Then which is(are) TRUE? a. $\exists x \in \mathbb{R}$ such that $f(x) = \frac{f(-1)+f(1)}{2}$ b. $\exists x \in \mathbb{R}$ such that $f(x) = \sqrt{f(-1)f(1)}$ c. $\exists x \in \mathbb{R}$ such that $f(x) = \frac{f^3(-1)+f^3(1)}{2}$ d. $\exists x \in \mathbb{R}$ such that $f(x) = [f(-1)f(1)]^{\frac{1}{3}}$	**CO1**
Q3	Let $f: \mathbb{R}\backslash\{0\} \to \mathbb{R}$ be defined by $f(x) = x + \frac{1}{x^3}$. On which of the following interval(s) is f one-one? a. $(-\infty, -1)$ b. $(0,1)$ c. $(0,2)$ d. $(0, \infty)$	**CO2**
Q4	Let $f: [0, \infty) \to [0, \infty)$ be continuous on $[0, \infty)$ and differentiable on $(0, \infty)$. If $f(x) = \int_0^x \sqrt{f(t)}\, dt$, then $f(6) =$ _____	**CO2**
Q5	The number of real roots of the equation $e^x - 2020x^2 = 0$ is ______	**CO3**
Q6	The coefficient of $\left(x - \frac{\pi}{2}\right)^{2019}$ in the Taylor's expansion of $\sin x$ about $x = \frac{\pi}{2}$ is _____	**CO3**

SECTION B

1. **Each question carries 10 marks.**
2. **There is an internal choice in Q11.**

Q7	Let $f:\mathbb{R}\to\mathbb{R}$ such that $f(x)=\sin x$ for all rational x; and $\cosh x$ otherwise. Find the accumulation point on $\mathbb{R}$ at which the limit of function exists.	**CO1**
Q8	Use the location of roots principle to find the smallest positive root of $e^x-3x^2=0$ correct to one decimal place.	**CO2**
Q9	Use $\epsilon-\delta$ definition to prove that the function x^3 is continuous on $\mathbb{R}$.	**CO2**
Q10	Check the uniform continuity of $\sin x^2$ on $(0,\infty)$.	**CO2**
Q11	Suppose f is differentiable on an interval $I\subset\mathbb{R}$ such that $f'(x)=0\ \forall x\in I$. Prove that f is constant on I. **OR** Prove that if f satisfies the Lipschitz condition of order $\alpha>1$ on some real interval I then f is constant on I.	**CO3**
SECTION-C 1. **Q12 carries (10+10) marks.** 2. **There is an internal choice in Q12.**		
Q 12	a. Suppose $f(x)$ is continuously differentiable at some point $a\in\mathbb{R}$ such that $$f(x)=\sum_{i=0}^{\infty}c_n(x-a)^n$$ Prove that the value of coefficient c_n is $\frac{f^n(a)}{n!}$. b. Use Maclaurin's expansion of $sin\,x$ to compute the value of $sin\,6°$ correct to two decimal places. **OR** a. Use Taylor's expansion to prove: $$sin(e^x-1)=x+\frac{x^2}{2}-\frac{5}{24}x^4+\cdots$$ b. Find the values of α and β such that the expansion of $log_e(1+x)-\frac{x(1+ax)}{1+bx}$ in ascending powers of x doesn't contain the terms up to degree 3.	**CO3**