Name: **Enrolment No:**	UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, July 2020

Course: Mathematics II **Semester: II**

Course Code: MATH 1027 **Time: 03 hrs.**

Programme: B.Tech. (All SoE Branches) **Max. Marks: 100**

Instructions: Attempt all questions from **PART A** (60 Marks) and **PART B** (40 Marks). All questions are compulsory.

PART A

Instructions: PART A contains 25 questions for a total of 60 marks. It contains 21 multiple-choice questions and 4 multiple answer questions. Multiple answer questions may have more than one correct option. Select all the correct options. You need to answer PART A within the slot from 10:00 AM to 1:00 PM on 6th July 2020. The due time for PART A is 1:00 PM on 6th July 2020. After the due time, the PART A will not be available.

S. No.		**Marks**	**CO**
Q1 (i)	The general solution of the differential equation $\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 15y = 0$ is A. $y = c_1e^{3x} + c_2xe^{-5x}$ B. $y = c_1e^{3x} + c_2e^{5x}$ C. $y = c_1xe^{-3x} + c_2e^{5x}$ D. $None\ of\ these$	**2**	**CO1**
Q1 (ii)	Particular integral of the differential equation $\frac{d^2y}{dx^2} + 4y = \cos 2x$ is A. $\frac{x}{4}\sin 2x$ B. $\frac{x}{2}\sin 2x$ C. $\frac{x}{4}\cos 2x$ D. $\frac{x}{2}\cos 2x$	**2**	**CO1**
Q1 (iii)	In solving $y'' + Py' + Qy = R$, if $P + Qx = 0$ then a part of the Complementary Function (C. F.) is A. x B. x^3 C. x^2 D. e^x	**2**	**CO1**

Q1 (iv)	If $f(z) = u(x, y) + iv(x, y)$ is an analytic function then $f'(z)$ equals A. $\frac{\partial v}{\partial y} - i\frac{\partial u}{\partial y}$ B. $\frac{\partial u}{\partial x} + 2\frac{\partial v}{\partial x}$ C. $\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}$ D. None of these	**2**	**CO2**
Q1 (v)	Value of the integration $\int_0^{2+i} (\bar{z})^2 dz$ along the line $y = \frac{x}{2}$ is A. $\frac{5}{3}(2 - i)$ B. $\frac{5}{3}(2 + i)$ C. $\frac{5}{3}(1 - i)$ D. $\frac{5}{3}(1 + i)$	**2**	**CO2**
Q1 (vi)	If $I = \oint_C \frac{\cos \pi z}{z^2 - 1}\, dz$ where C is a rectangle with vertices $2 \pm i, -2 \pm i$ then I is equal to A. -1 B. $2\pi i$ C. πi D. 0	**2**	**CO3**
Q1 (vii)	The transformation $w = \frac{az+b}{cz+d}$, where a, b, c and d are complex constants, is called the bilinear transformation if A. $ab - cd = 0$ B. $ab - cd \neq 0$ C. $ad - bc = 0$ D. $ad - bc \neq 0$	**2**	**CO3**
Q1 (viii)	Consider the function $f(z) = \frac{1}{(z-1)^2(z-3)}$. The residue of $f(z)$ at the singular point $z = 1$ is A. 0 B. $\frac{1}{2}$ C. $-\frac{1}{4}$ D. $-\frac{1}{2}$	**2**	**CO3**

Q1 (ix)	The radius of convergence of the power series $\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!} z^n$ is A. 2 B. $1/2$ C. 4 D. $1/4$	**2**	**CO3**
Q1 (x)	The nature of the singularity of the function $f(z) = \sin\frac{1}{1-z}$ at $z = 1$ is A. Removable Singularity B. Essential Singularity C. Pole of order 1 D. Pole of order 2	**2**	**CO3**
Q1 (xi)	The partial differential equation from the relation $u(x, y) = a(x + y) + b$, where a, b are arbitrary constants is A. $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$ B. $\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} = 0$ C. $\frac{\partial u}{\partial x} - 2\frac{\partial u}{\partial y} = 0$ D. $\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0$	**2**	**CO4**
Q1 (xii)	The solution of PDE $\frac{\partial^5 u}{\partial x^3 \partial y^2} - \frac{\partial^5 u}{\partial x^2 \partial y^3} = 0$ is A. $u = f_1(y) + x f_2(y) + f_3(x) + y f_4(x) + f_5(y + x)$. B. $u = f_1(-y) + f_2(-y) + f_3(x) - y f_4(x) + f_5(-y - x)$. C. $u = f_1(y) + x f_2(y) + f_3(x) + y f_4(x) + f_5(y + 3x)$. D. $u = f_1(2y) + x f_2(y) + f_3(x) + y f_4(x) + f_5(2y + x)$.	**2**	**CO4**
Q1 (xiii)	While solving the partial differential equation $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, with method of separation of variables we shall obtain A. One ordinary differential equation B. One ordinary and one partial differential equations. C. Two ordinary differential equations D. Two partial differential equations	**2**	**CO4**

Q1 (xiv)	The most general solution of the partial differential equation $u_{xx} = u_{tt}$, satisfying the boundary conditions $u(0,t) = u(1,t) = 0$ is A. $u(x,t) = \sum_{n=1}^{\infty} \sin n\pi x\,(A_n \cos n\pi t + B_n \sin n\pi t)$ B. $u(x,t) = \sum_{n=1}^{\infty} \cos n\pi x\,(A_n \cos n\pi t - B_n \sin n\pi t)$ C. $u(x,t) = \sum_{n=1}^{\infty} A_n \cos n\pi t \sin n\pi x$ D. $u(x,t) = \sum_{n=1}^{\infty} A_n \sin 4n\pi t \cos n\pi x$	**2**	**CO4**
Q1 (xv)	The partial differential equation corresponding to the arbitrary function $f(x^2 + y^2 + z^2, x + y + z) = 0$ is A. $(z-y)\frac{\partial z}{\partial x} - (x-z)\frac{\partial z}{\partial y} = py - x$ B. $(z-y)\frac{\partial z}{\partial x} + (x-z)\frac{\partial z}{\partial y} = y - x$ C. $(z-y)\frac{\partial z}{\partial x} + (x-zy)\frac{\partial z}{\partial y} = x - y$ D. $(z-y)\frac{\partial z}{\partial x} - x(x-z)\frac{\partial z}{\partial y} = x - y$	**2**	**CO4**
Q1 (xvi)	The general solution of the differential equation $\left(6x^2 - e^{-y^2}\right)dx + 2xye^{-y^2}dy = 0$ is A. $x^2\left(2x - e^{-y^2}\right) = c$ B. $x^2\left(2x + e^{-y^2}\right) = c$ C. $x\left(2x^2 - e^{-y^2}\right) = c$ D. $x\left(2x + e^{-y^2}\right) = c$	**3**	**CO1**
Q1 (xvii)	The value of n for which the differential equation $(3xy^2 + n^2x^2y)dx + (nx^3 + 3x^2y)dy = 0; x \neq 0$ be exact is (More than one answer can be correct) A. 3 B. 2 C. 1 D. 0	**3**	**CO1**
Q1 (xviii)	What is $f(z) = u + iv$ if $u = x^3 - 3xy^2$? A. $z^3 + c$ B. $3z^3 + c$ C. $z^2 + c$ D. $z^4 + c$	**3**	**CO2**

Q1 (xix)	If $f(t) = \int_C \frac{3z^2+7z+1}{z-t}\, dz$ where C is the circle $x^2 + y^2 = 4$ then which statements from the following are true? (More than one answer can be correct) A. $f(3) = 0$ B. $f(4) = 0$ C. $f(0) = 0$ D. $f(1) = 0$	**3**	**CO2**
Q1 (xx)	In the Taylor's series expansion of $\sin z$ about $z = \pi/4$, coefficient of $(z - \frac{\pi}{4})^2$ is A. 0 B. 1 C. $-\frac{1}{2\sqrt{2}}$ D. -$\frac{1}{\sqrt{2}}$	**3**	**CO3**
Q1 (xxi)	In which region from the following, the function $f(z) = 1/((z+1)(z+5))$ cannot be expanded in Laurent's series? A. $1 < \|z\| < 5$ B. $\|z\| < 1$ C. $\|z\| > 5$ D. None of these	**3**	**CO3**
Q1 (xxii)	Consider the integral $\int_C f(z)\, dz$, where $f(z) = \frac{1}{(z-1)(z+2)^2}$ and C is the circle given by $\|z\| = \frac{3}{2}$. Choose the correct statement(s). (More than one answer can be correct). A. $z = 1$ is the only singular point of $f(z)$ inside C. B. Residue of $f(z)$ at $z = 1$ is $-\frac{1}{9}$. C. Value of the integral is 0. D. Value of the integral is $\frac{2}{9}\pi i$.	**3**	**CO3**
Q1 (xxiii)	General solution of the PDE $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = xy$ is A. $f(x) = xe^{-1/xy}$ B. $f\left(x^2 e^{-\frac{u}{xy}}\right) = x$ C. $f\left(xy, xe^{-u/xy}\right) = 0$ D. $f(uy) = x^3 e^{-u/xy}$	**3**	**CO3**

Q1 (xxiv)	The solution of PDE $\frac{\partial^2 u}{\partial x^2} - 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = g(y+x)$ is A. $u = f_1(y-x) + xf_2(y-x) + \frac{x^2}{4}g(y+x)$ B. $u = f_1(y+x) + xf_2(y+x) + \frac{x^2}{2}g(y+x)$ C. $u = f_1(y-x) + f_2(y+x) + \frac{x^2}{4}g(y+x)$ **D.** $u = f_1(y-x) + f_2(y+x) + \frac{x^2}{2}g(y+x)$	**3**	**CO4**
Q1 (xxv)	The second order partial differential equation $u_{xx} + xu_{yy} = 0$ is A. Elliptic for $x > 0$ B. Hyperbolic for $x > 0$ C. Elliptic for $x < 0$ D. Hyperbolic for $x < 0$	**3**	**CO4**

PART B

The link for PART B will be available from 10:00 AM on 6th July 2020 to 10:00 AM on 7th July 2020. Solve the problems in PART B on a plain A4 sheets and write your name, roll number and SAP ID on each page and then scan them into a single PDF file. Name the file as SAP ID _BRANCH NAME_ROLL NUMBER (for example: 500077624_CCVT_ R103219023.pdf) and upload that PDF file through the link provided over there. PART B solutions sent through WhatsApp or email will not be entertained.

Q2	Solve the initial value problem $4\frac{d^2y}{dt^2} - y = 0; y(0) = 2, y'(0) = \beta.$ Then find β so that the solution approaches zero as $t \to \infty$.	**8**	**CO1**
Q3 (A)	For what value of the integer n the function $u(x,y) = x^n - y^n$ is harmonic?	**4**	**CO2**
Q3 (B)	Suppose that a function $f(z) = u(x,y) + iv(x,y)$ and its conjugate $\overline{f(z)} = u(x,y) - iv(x,y)$ are both analytic in a given domain D. Show that the function $f(z)$ must be constant through-out D.	**4**	**CO2**
Q4	Evaluate $\int_0^{2\pi} \frac{d\theta}{3 - 2\cos\theta + \sin\theta}$ using complex integration.	**8**	**CO3**
Q5	Find the integral surface of the linear first order partial differential equation $(x-y)p + (y-x-z)q = z,$ which passes through the circle $z = 1, x^2 + y^2 = 1$.	**8**	**CO4**
Q6 (A)	Discuss the nature of the singularity of the function $f(z) = \frac{\sin(z-a)}{(z-a)}$ at $z = a$.	**4**	**CO3**
Q6 (B)	Solve the partial differential equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} - 6\frac{\partial^2 z}{\partial y^2} = 0.$	**4**	**CO4**