UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2019

Course: Mathematical Methods   Semester: I
Course Code: DSQT1008

Programme: BA (Hons.) Specialization in Energy Economics
Time: 03 hrs.   Max. Marks: 100

Instructions: Answer **all** the questions from <u>Section A</u>, **Four** questions from <u>Section B</u>, **Two** questions from <u>Section C</u> and <u>Section D</u> is **compulsory.**

SECTION A (10*2 = 20 marks)

S. No.		Marks	CO
Q1	$A = \{2, 4, 6, 8, 10\}$, $B = \{1, 4, 5, 8\}$ and $C = \{1, 2, 4\}$. Find $A \cup B \cup C$, $A \cap B$ and $B \cap C$	2	1
Q2	$f(x) = x^2 + 1$. Find the value of the function for $f(-2)$, and $f(\sqrt{3})$.	2	1
	Find dy/dx for the following functions (Q3 to Q5)		
Q3	$y = (x^2 + 1)^3$	2	1
Q4	$y = (2x^2 - 2)(x + 1)$	2	1
Q5	$y = (x^2 - 1)/(x^3 + 2)$	2	1
	Find integration for the following functions (Q6 and Q7)		
Q6	$y = \int (x^3 - x^2)\, dx$	2	2
Q7	$y = \int_0^3 (2x^2 + x + 3)\, dx$	2	2
Q8	Find domain of the function $f(x) = \dfrac{x-3}{x^2-16}$	2	2
	Let $A = \begin{bmatrix} 1 & -2 \\ 2 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 \\ 3 & -2 \end{bmatrix}$	2	
Q9	Compute AB.	2	1
Q10	Compute $A + B$ and $B - A$	2	1

SECTION B (4*5 = 20 marks)

Q11	Find the rank of the following matrix. $B = \begin{bmatrix} 12 & 0 & 3 \\ 9 & 2 & 5 \\ 4 & 6 & 1 \end{bmatrix}$	5	1
Q12	Let the marginal cost (MC) function is given as $MC = 30 + 2Q - Q^2$, and fixed cost (FC) is 70. Find the total cost (TC), variable cost (VC) and average cost (AC) functions.	5	3

Q13	$y = -9x^2 + 72x - 13$ Find the critical values at which the function is either maximum/ minimum.	5	3
Q14	A survey in UPES revealed that 450 students read Times of India (ToI) and 300 students read Indian Express (IE). However, 50 students read both ToI and IE. How many students read only ToI? How many students read only IE? How many students have responded to the survey?	5	2
Q15	Use implicit differentiation to find the derivative *dy/dx* for the following equation. $4x^2 - y^3 = 74$	10	1
SECTION-C (2*15 = 30 marks)			
Q16	Use Cramer's rule to solve for the unknowns in the following system of equations. $$4x + y - 5z = 8$$ $$-2x + 3y + z = 12$$ $$3x - y + 4z = 5$$	15	2
Q17	The total cost function is given as $C(x) = x^3 - 5x^2 + 60x, x \geq 0$, where x represents units of output. (a) Compute the marginal cost function $C'(x)$. (b) Find the value of x at which average cost (AC) is minimum.	15	3

Q18	Assume that the total revenue function is $TR = 1400Q - 7.5Q^2$ and total cost function is $TC = Q^3 - 6Q^2 + 140Q + 750$, and $Q > 0$.
	     (a) Find the value of output at which profit is maximum.
	     (b) Calculate the maximum profit.
		15	4

SECTION-D (30 marks)			
Q19	Assume that the utility function is given as $U(x, y) = 4x^2 + 3xy + 6y^2$. Price per unit of good x and good y is Re. 1 and the consumer has income of Rs. 56. Find the optimum bundle of x and y at which the consumer's utility/satisfaction is maximum.	30	4