

Name:  
Enrolment No:



**UNIVERSITY OF PETROLEUM AND ENERGY STUDIES**  
**End Semester Examination, December 2019**

**Course: Applied Numerical Methods**  
**Program: B. Tech. EE**  
**Course Code: MATH 306**

**Semester: VII**  
**Time 03 hrs.**  
**Max. Marks: 100**

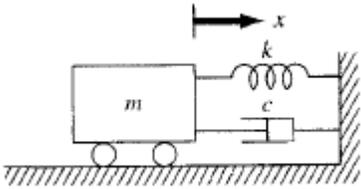
**Instructions:** All questions are compulsory. Internal choice is visible in the question(s). Calculator is allowed.

**SECTION A**

S. No.		Marks	CO								
Q 1	Write general form of a second order linear Partial Differential Equation. Classify it in Parabolic, Elliptic and Hyperbolic equations.	4	CO3								
Q 2	Evaluate $\int_0^1 \frac{dx}{1+x^2}$ using Simpson's $\frac{3}{8}$ rule taking $h = \frac{1}{6}$ .	4	CO4								
Q 3	Find a positive value of $(17)^{\frac{1}{3}}$ correct to four decimal places by Newton-Raphson method.	4	CO1								
Q 4	Using Lagrange's formula for interpolation find the value of y for x = 9.5 for a function y = f(x) which has following set of values.  <table border="0" style="margin-left: auto; margin-right: auto;"> <tr> <td>x:</td> <td>8</td> <td>9</td> <td>10</td> </tr> <tr> <td>y:</td> <td>1</td> <td>1</td> <td>9</td> </tr> </table>	x:	8	9	10	y:	1	1	9	4	CO4
x:	8	9	10								
y:	1	1	9								
Q 5	Prove that $e^x = \left(\frac{\Delta^2}{E}\right) e^x \frac{Ee^x}{\Delta^2 e^x}$ , where symbols have their usual meanings.	4	CO4								

**SECTION B**

Q 6	Use Euler's modified method to compute y for x = .05 and x = 0.1 given $\frac{dy}{dx} = x + y$ with initial condition $x_0 = 1, y_0 = 1$ . Give the result correct to two places of decimal.	10	CO2										
Q 7	Find the first derivative of the function tabulated below at the point $x = 1.1$ , using numerical differentiation.  <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>x</td> <td>1.0</td> <td>1.2</td> <td>1.4</td> <td>1.6</td> </tr> <tr> <td>f(x)</td> <td>0</td> <td>.1280</td> <td>.5440</td> <td>1.2960</td> </tr> </table>	x	1.0	1.2	1.4	1.6	f(x)	0	.1280	.5440	1.2960	10	CO4
x	1.0	1.2	1.4	1.6									
f(x)	0	.1280	.5440	1.2960									

Q 8	<p>The mean atmospheric refraction R, for a star at various altitudes <math>h^\circ</math> above the horizon is given in the table below. Using forward difference interpolation formula, find the refraction for a star at an altitude of <math>23^\circ</math> above the horizon.</p> <table border="1" data-bbox="298 411 1140 529"> <tbody> <tr> <td><b>H</b></td> <td><math>22^\circ</math></td> <td><math>24^\circ</math></td> <td><math>26^\circ</math></td> <td><math>28^\circ</math></td> </tr> <tr> <td><b>R</b></td> <td><math>2' 23''.3</math></td> <td><math>2' 10''.2</math></td> <td><math>1' 58''.9</math></td> <td><math>1' 49''.2</math></td> </tr> </tbody> </table>	<b>H</b>	$22^\circ$	$24^\circ$	$26^\circ$	$28^\circ$	<b>R</b>	$2' 23''.3$	$2' 10''.2$	$1' 58''.9$	$1' 49''.2$	10	CO4
<b>H</b>	$22^\circ$	$24^\circ$	$26^\circ$	$28^\circ$									
<b>R</b>	$2' 23''.3$	$2' 10''.2$	$1' 58''.9$	$1' 49''.2$									
Q 9	<p>Find a positive real root of <math>x - \cos x = 0</math> between 0 and 1 by regula - falsi method correct up to 2 decimal places.</p> <p style="text-align: center;"><b>OR</b></p> <p>Apply Graeffe's root squaring method to solve the equation <math>x^3 - 8x^2 + 17x - 10 = 0</math>, squaring twice.</p>	10	CO1										
<b>SECTION-C</b>													
Q 10A	<p>Solve the following equations by Gauss Seidel iteration method correct up to 2 decimal places.</p> $20x + y - 2z = 17 ; 3x + 20y - z = -18 ; 2x - 3y + 20z = 25$	10	CO1										
Q 10B	<p>The motion of a damped spring- mass system shown in the following figure is described by the differential equation <math>m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + kx = 0</math>, where x is the displacement from the equilibrium position (m), t is time in seconds, <math>m=10</math> kg is mass, and c is the damping coefficient which takes values 5 (under damped) and 40 (critically damped). The spring constant <math>k = 40</math> N/m.</p> <div style="text-align: center;">  </div> <p>The initial velocity is zero and the initial displacement <math>x=1</math> m. Solve this system and compare the displacements at <math>t=1</math> s for under damped as well as critically damped conditions with step size <math>t = 0.5</math> s.</p>	10	CO2										
Q 11	<p>Solve <math>u_{xx} + u_{yy} = 0</math> in <math>0 \leq x \leq 4, 0 \leq y \leq 4</math> with the given conditions <math>u(0, y) = 0; u(4, y) = 8 + 2y; u(x, 0) = x^2/2</math> and <math>u(x, 4) = x^2</math> by taking <math>h = k = 1</math>. (Obtain the result correct to one place of decimal.)</p> <p style="text-align: center;"><b>OR</b></p>	20	CO3										

Using Crank – Nicholson method, solve $u_{xx} = 16 u_t$ , $0 < x < 1$ , $t > 0$ given $u(x, 0) = 0$ , $u(0, t) = 0$ and $u(1, t) = 50t$ . Compute $u$ for two steps in $t$ direction tacking $h = \frac{1}{4}$ .		
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