Name:

Enrolment No:

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2019

Course: Finite Element Methods **Semester: VII**

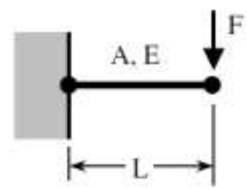
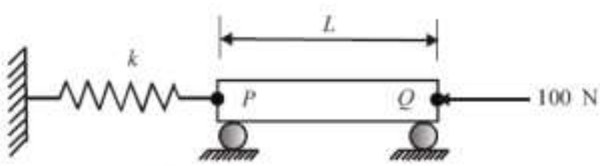
Program: B. Tech Aerospace Engineering **Time 03 hrs.**

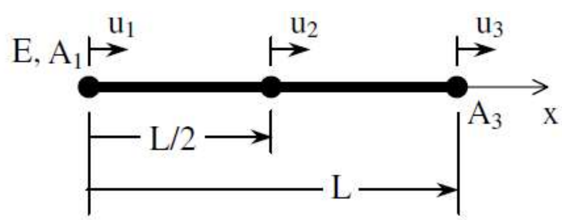
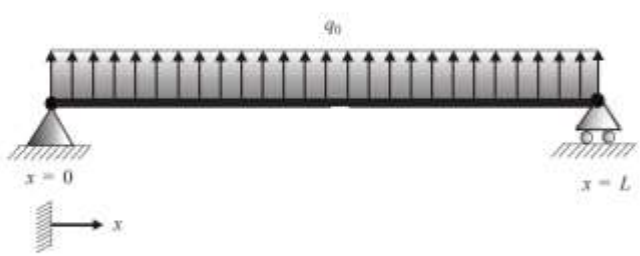
Course Code: ASEG 417 **Max. Marks: 100**

No. of pages:04

Instructions: Make use of sketch/plots to elaborate your answer. All sections are compulsory

SECTION A (20 marks)

S. No.		Marks	CO
Q 1	Consider the horizontal BAR element with cantilevered support conditions. Is the stiffness matrix singular (i.e. can you solve for the displacement)? 	**[04]**	**CO1**
Q 2	Given the following stress tensor $$\sigma = \begin{bmatrix} 10 & 20 & 30 \\ 20 & 40 & 50 \\ 30 & 50 & 60 \end{bmatrix}$$ I. What is the value of von Mises stress? II. Propose two other stress tensor that will have the same von Mises stress? III. Do all stress tensors having the same von Mises stress also have the same principle stresses? IV. Do all stress tensors having the same principle stresses also have the same von Mises stress?	**[04]**	**CO2**
Q 3	Solve the following equation using a two-parameter trial solution by the Rayleigh-Ritz method, $$\frac{dy}{dx} + y = 0, \qquad y(0) = 1$$	**[04]**	**CO2**
Q 4	Consider the spring-mounted bar as shown in the figure. Solve for the displacements of points P and Q using the bar elements (assume AE = constant). 	**[04]**	**CO2**
Q 5	Define types of elements with proper schematic for different dimensions of space. Give examples for each type of elements.	**[04]**	**CO1**

SECTION B (40 marks)			
Q 6	Given the four-dimensional vectors $x = \begin{bmatrix} 2 \\ 4 \\ 4 \\ -8 \end{bmatrix}, \qquad y = \begin{bmatrix} 21 \\ -5 \\ 7 \\ -5 \end{bmatrix}$ (a) Compute the Euclidean norms and lengths of **x** and **y**; (b) Compute the inner product (**x**,**y**); (c) Verify that the inequalities |(**x**, **y**)| ≤ |**x**| |**y**|, and |**x** + **y**| ≤ |**x**| + |**y**| holds; (d) Normalize **x** and **y** to unit length	**[10]**	**CO1**
Q 7	A 3 node rod element has a quadratic shape function matrix: $N = \langle 1 - \frac{3x}{L} + \frac{2x^2}{L^2}, \frac{4x}{L} - \frac{4x^2}{L^2}, -\frac{x}{L} + \frac{2x^2}{L^2} \rangle$ For $L = 1\ m$, $E = 200 \times 10^9$ Pa, $u_1 = 0, u_2 = 5 \times 10^{-6} m, u_2 = 15 \times 10^{-6} m$ Find: a. The displacement u at $x = 0.25\ m$. b. The strain as a function of x. c. The strain at $x = 0.25\ m$. d. The stress at $x = 0.25\ m$  	**[10]**	**CO4**
Q 8	Consider a simply supported beam under uniformly distributed load q_0 as shown in figure. For the deformation $v(x)$, we have  Find the appropriate approximation for deformation $v(x)$ using the Principle of Stationary Total Potential (PSTP).	**[10]**	**CO3**

Q 9	Considering the following force displacement relationship,	[10]	CO4

$$\mathbf{f} = \mathbf{K}\mathbf{u} = \begin{bmatrix} 20 & 10 & -10 & 0 & -10 & -10 \\ 10 & 10 & 0 & 0 & -10 & -10 \\ -10 & 0 & 10 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & -5 \\ -10 & -10 & 0 & 0 & 10 & 10 \\ -10 & -10 & 0 & -5 & 10 & 15 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0.4 \\ -0.2 \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \\ 0 \\ 1 \\ 2 \\ 1 \end{bmatrix}$$

Draw a free body diagram of the nodal forces acting on the free-free truss structure and verify that this force system satisfies translational and rotational (moment) equilibrium.

OR

Solve the differential equation for a physical problem expressed as $\frac{d^2y}{dx^2} + 100 = 0$

$0 \leq x \leq 10$ with boundary conditions as y(0)=0 and y(10)=0 using

(i) Point collocation method
(ii) Sub domain collocation method

SECTION-C (40 marks)

Q 10	Derive the Euler-Lagrange equation for a functional given by,	[20]	CO4

$$I(u) = \int_a^b F\left(u, \frac{du}{dx}, x\right) dx$$

Thus, obtain the corresponding Euler-Lagrange for the functional given below,

$$I = \frac{1}{2}\int_0^L \left[\propto \left(\frac{dy}{dx}\right)^2 - \beta y^2 + r y x^2\right] dx - y(L)$$

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<td>Q 11</td>
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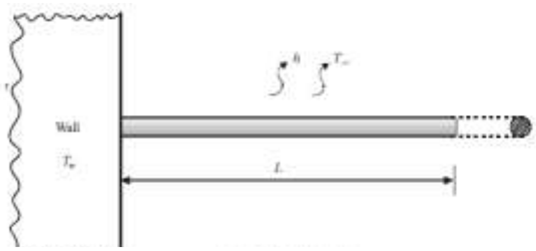
Consider a 1 *mm* diameter, 50 *mm* long aluminum pin fin as shown in the figure below that is used to enhance the heat transfer from a surface wall maintained at 300°C. The governing differential equation and the boundary conditions are given by,

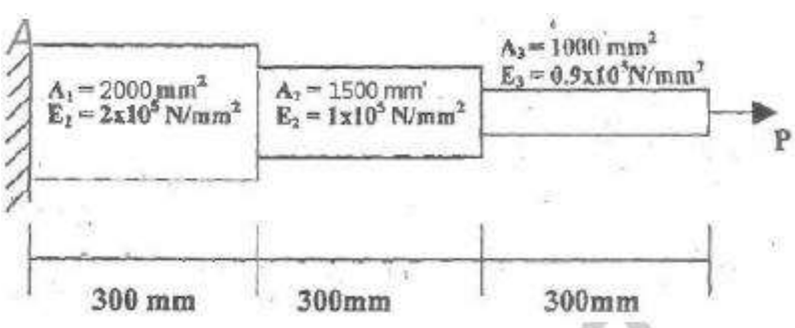


$$k\frac{d^2T}{dx^2} = \frac{Ph}{A_c}(T - T_\infty); \qquad T(0) = T_w = 300^oC, \qquad \frac{dT}{dx}_{(L)} = 0$$

Let k = 200W/m/°C for aluminum, h = 20 W/m^{2o}C, T_∞ = 30°C. Estimate the temperature distribution in the fin at 10 equal points using the Galerkin residual method using an appropriate polynomial trial function.

OR

Consider the bar shown in figure axial force P = 30KN is applied as shown. Determine the nodal displacement, stresses in each element and reaction forces.



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<td>[20]</td>
<td>CO5</td>
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