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UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2019

Programme: BSc(Hons)Mathematics | **Semester – III**
Course Name: Group Theory-1 | **Max. Marks : 100**
Course Code: MATH 2028 | **Duration : 3 Hrs**
No. of page/s:2

Instructions: The multiple sub-parts of a question must be answered together.

Section A
(Attempt all questions)

		MARKS	
1.	If H is a subgroup of G, then show that $\bigcap_{g\in G} gHg^{-1}$, is a normal subgroup of G.	**[4]**	**CO1**
2.	Show that every homomorphic image of a cyclic group is cyclic.	**[4]**	**CO1**
3.	Find $Aut(G)$, if $G =< a >,\ a^{12} = e$.	**[4]**	**CO1**
4.	Determine the permutation $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 5 & 4 & 3 & 6 & 1 & 9 & 7 & 8 \end{pmatrix}$ is even or odd.	**[4]**	**CO2**
5.	Show that the quaternion group $G = \{\pm 1, \pm i, \pm j, \pm k\}$, where $i^2 = j^2 = k^2 = -1$, $ij = -ji = k,\ jk = -kj = i,\ ki = -ik = j$ cannot expressed as the internal direct product of its proper subgroups.	**[4]**	**CO2**

SECTION B
(All questions are compulsory, Q10 has internal choice)

6.	Let N be a normal subgroup of G of finite index and H be a subgroup of G of finite order such that $[G : N]$ is relatively prime to $O(H)$. Prove that $H \subseteq N$.	**[08]**	**CO3**
7.	Suppose G is an abelian group. If $a, b \in G$ such that $O(a) = m, O(b) = n$ and $(m, n) = 1$ then prove that $O(ab) = mn$.	**[08]**	**CO1**
8.	Let S_n be a symmetric group of n symbols and let A_n be the group of even permutations. Then show that A_n is normal in S_n and $O(A_n) = \frac{n!}{2}$.	**[08]**	**CO3**

9.	Show that a group of order 4 is either cyclic or is an internal direct product of two cyclic subgroups each of order 2.	**[08]**	**CO5**
10.	Show that the set $\mathbb{Q}^+$ of all positive rational numbers is an abelian group under the binary operation $*$, defined by $a * b = \frac{ab}{3},\ \forall a, b \in \mathbb{Q}^+$. **OR** If G is a group then show that the normalizer of an element in G is a subgroup of G.	**[08]**	**CO2**
SECTION C **(Q11 is compulsory and Q12 has internal choice)**			
11.A	Show that a group G of order $2p$, where p is prime and $p > 2$, has exactly one subgroup of order p.	**[10]**	**CO4**
11.B	Show that the set G of all symmetries of a rectangle is a group and hence find a Klein 4-group which is a subgroup of G.	**[10]**	**CO2**
12	State and prove the second isomorphism theorem of groups. **OR** Let G be the dihedral group defined as $G = \{x^i\, y^j :\ i = 0, 1; j = 0,1,2, \dots, n-1; where\ x^2 = e, y^n = e, xy = y^{-1}x\}$. Prove that (i) The subgroup $N = \{e, y, y^2, \dots, y^{n-1}\}$ is normal in G. (ii) $\frac{G}{N} \cong W$ where $W = \{1, -1\}$ is the group under the multiplication of the real numbers.	**[20]**	**CO5**

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