Name: Enrolment No:	UPES UNIVERSITY WITH A PURPOSE

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2019

Course : Differential Equations — **Semester : III**
Program : B.Sc. (Honours) (Physics/Chemistry) — **Time : 03 hrs.**
Course Code: MATH 1034 — **Max. Marks: 100**

Instructions:
Attempt all questions from Section A (each carrying 4 marks); attempt all questions from Section B (Each carrying 10 marks) and attempt all questions from Section C (each carrying 20 marks). Question 9 and 11 have internal choice.

SECTION A

S. No.		Marks	CO
Q 1	Use the Wronskian to prove that the functions $f(x) = e^x$, $g(x) = e^{2x}$ and $h(x) = e^{3x}$ are linearly independent on the real line.	**4**	**CO1**
Q 2	Solve the differential equation $$\sin px \cos y = \cos px \sin y + p \text{ where } p = \frac{dy}{dx}.$$	**4**	**CO2**
Q 3	Construct a linear homogeneous ordinary differential equation with constant coefficients whose general solution is given by $$x(t) = (c_1 + c_2 t + c_3 t^2)\cos 2t + (c_4 + c_5 t + c_6 t^2)\sin 2t,$$ where c_1, c_2, c_3, c_4, c_5 and c_6 are arbitrary constants.	**4**	**CO3**
Q 4	Find the solution of the following simultaneous differential equations: $$Dx - y = 0,$$ $$-x + Dy = 1,$$ where $D \equiv \frac{d}{dt}$.	**4**	**CO4**
Q 5	Show that the equation $u_{xx} + xu_{yy} + u_y = 0$ is elliptic for $x > 0$ and hyperbolic for $x < 0$.	**4**	**CO5**

SECTION B

S. No.		Marks	CO
Q 6	Form a partial differential equation by eliminating the arbitrary functions f and g from $z = f(x^2 - y) + g(x^2 + y)$.	**10**	**CO1**
Q 7	Consider the initial value problem $$y' = \frac{-3x^2y^4}{4x^3y^3}, y(1) = 1.$$ Find the solution of the initial value problem in implicit form.	**10**	**CO2**

Q 8	Find the general solution of the differential equation $y'' + 16y = 32\sec 2x$, using the method of variation of parameters.	**10**	**CO3**
Q 9	Solve the differential equation $$\frac{dx}{x^2+2y^2} = \frac{dy}{-xy} = \frac{dz}{xz}.$$ **OR** Find $f(y)$ such that the total differential equation $$\{(yz+z)/x\}dx - zdy + f(y)dz = 0,$$ is integrable. Hence, solve it.	**10**	**CO4**
SECTION-C			
Q 10	**(i)** Solve the Cauchy-Euler homogeneous differential equation $$x^3\frac{d^3y}{dx^3} + 5x^2\frac{d^2y}{dx^2} + 5x\frac{dy}{dx} + y = x^2 + \ln x,\ x > 0.$$ **(ii)** Find the general solution of the differential equation $$(D^2+2)y = e^x\cos x + x^2e^{2x},$$ where $D \equiv \frac{d}{dx}$.	**10+10**	**CO3**
Q 11	**(i)** Solve the partial differential equation $$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x\partial y} - 2\frac{\partial^2 z}{\partial y^2} = (2x^2+xy-y^2)\sin xy - \cos xy.$$ **(ii)** Find the general solution of the partial differential equation $$(3-2yz)p + x(2z-1)q = 2x(y-3).$$ Hence, obtain the particular solution that passes through the curve $z = 0, x^2+y^2 = 4$. **OR** **(i)** Solve the partial differential equation $$q + xp - p^2 = 0 \text{ where } p = \frac{\partial z}{\partial x} \text{ and } q = \frac{\partial z}{\partial y},$$ by Charpit's method. **(ii)** Find the complete solution of the partial differential equation $$(D^2 - D'^2 - 3D + 3D')z = xy + e^{x+2y},$$ where $D \equiv \frac{\partial}{\partial x}$ and $D' \equiv \frac{\partial}{\partial y}$.	**10+10**	**CO5**