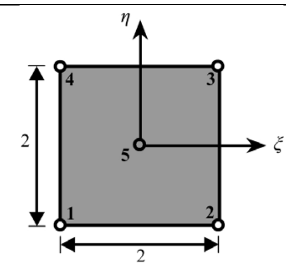
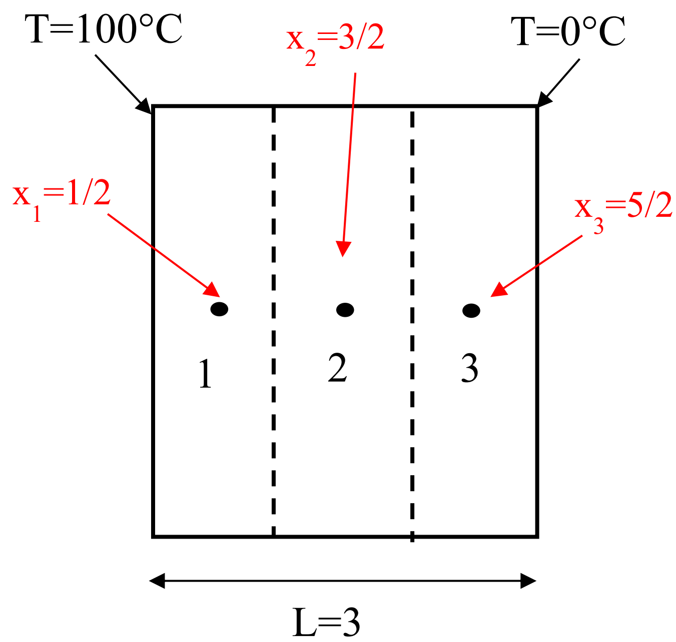
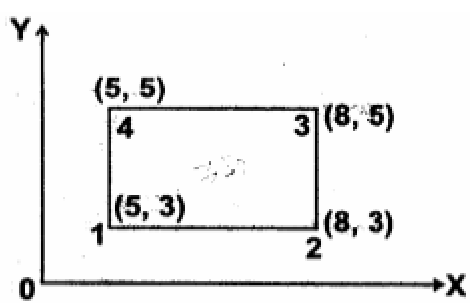
Name: **Enrolment No:**	UPES UNIVERSITY WITH A PURPOSE

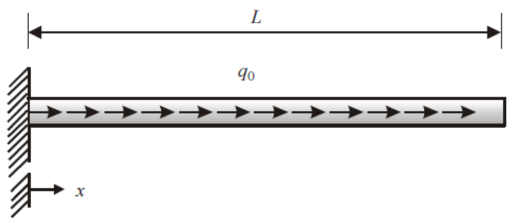
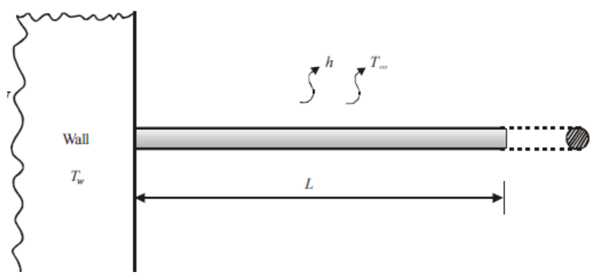
UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, December 2019

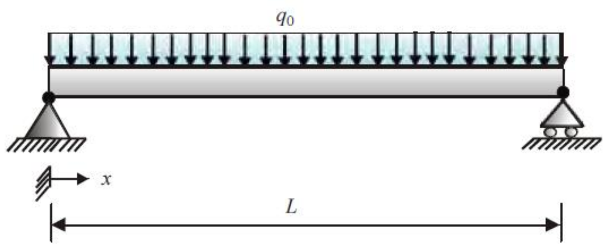
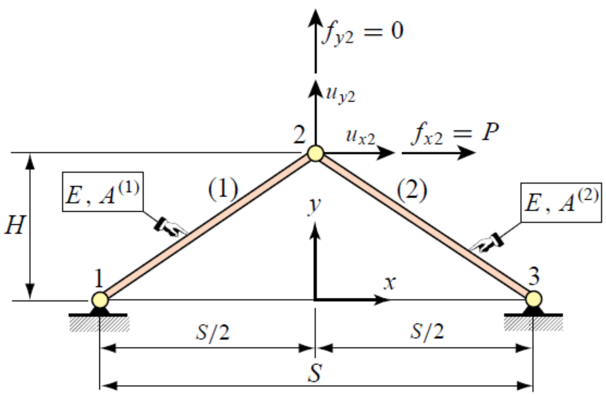
Course: Finite Element Methods for Fluid Dynamics **Semester: I**
Program: M. Tech CFD **Time 03 hrs.**
Course Code: ASEG 7022 **Max. Marks: 100**
No.of pages:05
Instructions: Make use of sketch/plots to elaborate your answer. All sections are compulsory

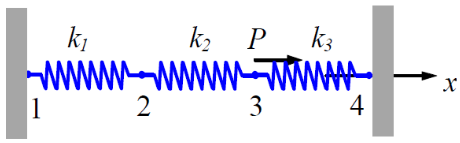
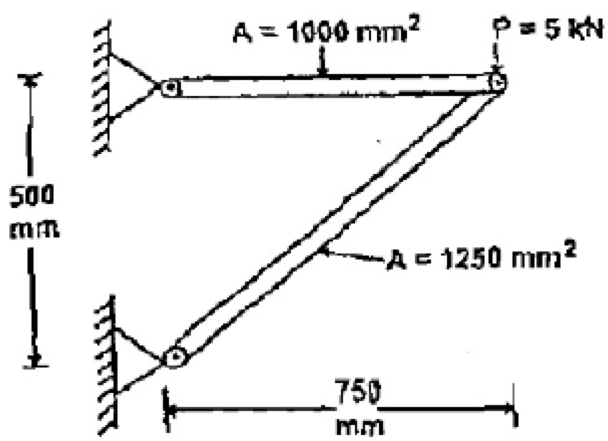
SECTION A (20 marks)

S. No.		**Marks**	**CO**
Q 1	State the finite element equation for a two dimensional triangular element placed in the Cartesian coordinate with origin on one side of the element.	**[04]**	**CO1**
Q 2	Determine the shape functions for the five-node rectangular element shown in the fig. 	**[04]**	**CO2**
Q 3	Explain the various shapes of finite elements that can be utilized with classification for one, two and three dimensional elements. Sketch clearly giving details of the corner and side nodes.	**[04]**	**CO1**
Q 4	Given the following stress tensor $$\sigma = \begin{bmatrix} 10 & 20 & 30 \\ 20 & 40 & 50 \\ 30 & 50 & 60 \end{bmatrix}$$ Calculate the traction vector on a plane with unit normal $\boldsymbol{n} = (0.100, 0.700, 0.707)$	**[04]**	**CO2**
Q 5	Show that starting with: $$\epsilon_{ijk}\epsilon_{lmn}=\delta_{il}(\delta_{jm}\delta_{kn}-\delta_{jn}\delta_{km})+\delta_{im}(\delta_{jn}\delta_{kl}-\delta_{jl}\delta_{kn})+\delta_{in}(\delta_{jl}\delta_{km}-\delta_{jm}\delta_{kl})$$ and multiplying both sides by δ_{il} produces:	**[04]**	**CO2**

	$\epsilon_{ijk}\epsilon_{imn}=\delta_{jm}\delta_{kn}-\delta_{jn}\delta_{km}$		
SECTION B (40 marks)			
Q 6	Determine the temperature distribution of the flat plate as shown below using finite element analysis. Assume one-dimensional heat transfer, steady state, no heat generation and constant thermal conductivity. The two surfaces of the plate are maintained at constant temperatures of 100°C and 0°C, respectively. 	**[10]**	**CO3**
Q 7	For a 4-noded rectangular element shown in fig. Calculate the temperature at point (7, 4). The nodal values of the temperatures are T1 = 42°C, T2 = 54°C and T3 = 56°C and T4 = 46°C. Also determine 3 point on the 50°C contour line. All dimensions are in cm. 	**[10]**	**CO4**
Q 8	Consider a uniform rod subjected to a uniform axial load as shown in fig. The deformation of the bar is governed by the differential equation; $AE\frac{d^2u}{dx^2}+q_0=0,$ and the boundary conditions; $u(0)=0,\ \frac{du}{dx}_{x=L}=0.$	**[10]**	**CO3**

	 		
Q 9	Consider a 1 *mm* diameter, 50 *mm* long aluminum pin fin as shown in the fig. that is used to enhance the heat transfer from a surface wall maintained at 300°C. The governing differential equation and the boundary conditions are given by, $k\frac{d^2T}{dx^2} = \frac{Ph}{A_c}(T - T_\infty);$ $T(0) = T_w = 300^oC,$ $\frac{dT}{dx}_{(L)} = 0$ Let k = 200W/m/°C for aluminum, h = 20 W/m^2°C, T_∞ = 30°C. Estimate the temperature distribution in the fin at 10 equal points using the Galerkin residual method using an appropriate polynomial trial function.  **OR** Consider a simply supported beam under uniformly distributed load as shown in fig. The governing differential equation and the boundary conditions are given by; $EI\frac{d^4v}{dx^4} - q_0 = 0;$ $v(0) = 0, \frac{d^2v}{dx^2}(0) = 0, v(L) = 0, \frac{d^2v}{dx^2}(L) = 0$ Find the approximate solution using the point collocation technique at $x = L/2$.	**[10]**	**CO3**

			
SECTION-C (40 marks)			
Q 10	Consider the two-member arch-truss structure shown in figure. Take span S = 8, height H = 3, elastic modulus E = 1000, cross section areas $A^{(1)}$ = 2 and $A^{(2)}$ = 4, and horizontal crown force $P = f_{x2}$ = 12.  Using the DSM carry out the following steps: (i) Assemble the master stiffness equations. (ii) Apply the displacement BCs and solve the reduced system for the crown displacements u_{x2} and u_{y2}. (iii) Recover the node forces at all joints including reactions. Verify that overall force equilibrium (x forces, y forces, and moments about any point) is satisfied. (iv) Recover the axial forces in the two members.	**[20]**	**CO5**
Q 11	For the spring system shown below, k_1 = 200 N / mm, k_2 = 100 N / mm, k_3 = 200 N / mm	**[20]**	**CO5**

	P = 10 N (applied at point 3). The fixed boundary leads to the displacement $U_1 = U_4 = 0$  *Find*: (a) Global stiffness matrix (b) Displacements of nodes 2 and 3 (c) Reaction forces at nodes 1 and 4 (d) Force in the spring 2 **OR** The loading and other parameters for a two bar truss element is shown in figure. Assume E=200GPa  Calculate (i) The element stiffness matrix for each element (ii) Global stiffness matrix (iii) Nodal displacements (iv) Reaction force (v) The stresses induced in the elements.		