Name: **Enrolment No:**	UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2019

Programme Name: M. Tech. CFD — **Semester : I**
Course Name : Computational Gas Dynamics — **Time : 03 hrs.**
Course Code : ASEG 7020 — **Max. Marks: 100**
Nos. of page(s) : 03
Instructions: Assume any missing data appropriately.

SECTION A

S. No.		Marks	CO
Q 1	Discuss the condition on wave speeds for the occurrence of an expansion wave in a one-dimensional space.	**4**	**CO1**
Q 2	Find the conservative numerical flux $f^n_{i+1/2}$ of the Roe's first order upwind method.	**4**	**CO3**
Q 4	Using the exact Riemann solver, write expressions for the fluxes at the cell interface $A\boldsymbol{u}(x=0)$ in terms of left and right states for various wave speeds.	**4**	**CO2**
Q 4	Project a first order upwind method for the linear advection equation using wave speed splitting.	**4**	**CO3**
Q 5	The unsteady Euler Equations have a full wave description. Justify	**4**	**CO1**

SECTION B

Q 6	For a Roe's approximate Riemann problem for the Euler problem is given as $$\frac{\partial \boldsymbol{u}}{\partial t} + A_{RL}\frac{\partial \boldsymbol{u}}{\partial x} = 0$$ where $$\boldsymbol{u}(x,0) = \begin{cases} \boldsymbol{u}_L & x < 0 \\ \boldsymbol{u}_R & x > 0 \end{cases}$$ Calculate the Roe-average velocity at the cell interface in terms of the velocities and	**10**	**CO3**

	densities on the left and right of the cell interface.		
Q 7	Show that, for an isothermal flow, the one dimensional unsteady Euler equations can be written as $$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) = 0$$ $$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho(u^2 + a^2)) = 0$$ ***OR*** Consider the one dimensional Euler Equations $$\frac{\partial \mathbf{u}}{\partial t} + A\frac{\partial \mathbf{u}}{\partial x} = 0,$$ Show that the Jacobian Matrix A is diagonalizable, i.e. $Q_A^{-1} A Q_A = \Lambda$.	**10**	**CO1**
Q 8	Prove that Van Leer's flux vector splitting satisfies $d\boldsymbol{f}^{+}/d\boldsymbol{u} \geq 0$ and $d\boldsymbol{f}^{-}/d\boldsymbol{u} \leq 0$.	**10**	**CO4**
Q 9	For a steady state adiabatic flow, assuming *s=const.*, and $u+2a/(\gamma-1)=const.$, derive an expression for the velocity *u*, speed of sound *a* and pressure *p* in the expansion fan centered on (*x*, *t*)=(0,0), which connects two steady uniform flows $\boldsymbol{u}_L$ and $\boldsymbol{u}_R$, as a function of space and time.	**10**	**CO2**
	SECTION-C		
Q 10	Find the solution to Roe's approximate Riemann problem at *t*= 0.01 s if p_L= 100, 000 N/m^2, ρ_L=1 kg/m^3, u_L=100 m/s and p_R=10,000 N/m^2, ρ_R = 0.125 kg/m^3, u_R=-50 m/s.	**20**	**CO3**
Q 11	The 1-D unsteady Euler equations are given by $$\boldsymbol{U}_t + [A]\boldsymbol{U}_x = 0$$ where $$\boldsymbol{U} = [\rho, u, p]^T$$	**20**	**CO4**

and

$$[A] = \begin{bmatrix} u & \rho & 0 \\ 0 & u & {}^{1}/_{\rho} \\ 0 & \rho a^2 & u \end{bmatrix}$$

Find the eigenvalues and the left eigenvectors for this system of equations.

OR

Apply Roe’s scheme to the following system of equations,

$$\boldsymbol{U}_t + [A]\boldsymbol{U}_x = 0$$

where

$$\boldsymbol{U} = [\rho, u, p]^T$$

and

$$[A] = \begin{bmatrix} u & \rho & 0 \\ 0 & u & {}^{1}/_{\rho} \\ 0 & \rho a^2 & u \end{bmatrix}$$

and thus evaluate the Roe-averaged Jacobian matrix $[|A|]=[T][|\Lambda|][T]^{-1}$, if $0 < u < a$.