Name:

Enrolment No:

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2019

Programme Name: M. Tech (Pipeline Engineering) **Semester: 1**
Course Name : Numerical Methods in Engineering
Time : 03 hours
Course Code : CHPL7003 **Max. Marks: 100**
Nos. of page(s) : 03

Instructions:

i. Use of scientific calculator is allowed for calculations.
ii. Any pages used for rough work should be attach along with the answer script.

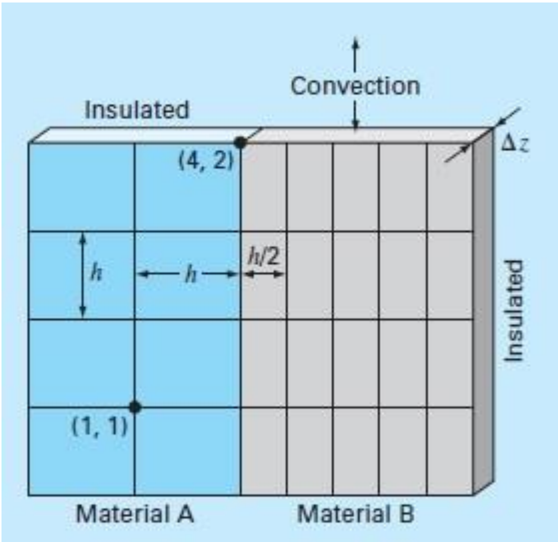
SECTION A

S. No.		Marks	CO
Q 1	Write the general expression for **3rd** order Newton's interpolating polynomial.	**4**	**CO1**
Q 2	State the difference between LU decomposition with Gauss elimination method and Crout's decomposition method.	**4**	**CO2**
Q 3	State the differences between bracketed and open method for finding the root (s) of an equation.	**4**	**CO3**
Q 4	What do you mean by 'stiffness' of ordinary differential equations?	**4**	**CO4**
Q 5	What is the difference between Dirichlet and Neumann boundary condition?	**4**	**CO5**

SECTION B

Q 6	Use any step size, *h*, and numerically integrate the following using **(i)** trapezoidal method, and **(ii)** Simpson's 1/3 rule to: $$\int_0^6 \frac{1}{1+x^2}dx$$ **OR** Use **2nd** order Lagrange interpolating polynomials to evaluate the value of *f*(*x*) at ***x* = 2**, from the following data given, *X* = 1, 4, 6; *f(x)* = 0, 0.60206, 0.7782	**8**	**CO1**

X	*f(x)*
1	0
4	0.60206
6	0.7782

Q 7	Use **Gauss – Jordan** method to solve the following simultaneous linear equations: $3x_1 + 4x_2 + x_3 = 26$ $x_1 + 2x_2 + 6x_3 = 22$ $6x_1 - x_2 - x_3 = 19$ Check your answers by substituting them into the original equations.	**8**	**CO2**
Q 8	Determine the real roots of the function, $f(x) = 4x^3 - 6x^2 + 7x - 2.3$, using **false position** method to locate the roots. Employ an **initial guess** of, x_l = 0, and x_u = 1 and make **3** iterations and calculate the approximate error, ε_a for each iteration.	**8**	**CO3**
Q 9	Use **Euler's method** to numerically solve the following differential equation, $\frac{dy}{dx} = -2x^3 + 12x^2 - 20x + 8.5$ From x = 0 to x = 4, with a step size (h) of **1**. The initial condition at x = 0 is y = 1. If the true value of the solution at x = 4 is **3.0000**. Find the true error at x = 4.	**8**	**CO4**
Q 10	Use the **control-volume** approach and derive the **node equation** for node (2, 2) in **Fig. 1**, and include a heat source at this point. The following constants are given as: Δz = 0.25 cm, h = 10 cm, k_A = 0.25 W/cm ·C, and k_B = 0.45 W/cm ·C. The heat source comes only from material **A** at the rate of 6 W/cm^3.  **Fig 1:** A heated plate with unequal spacing, two materials, and mixed boundary conditions.	**8**	**CO5**
SECTION-C			
Q 11	Find the **first** and **second** derivative of the following tabulated data at the point, x = 4 using (**i**) forward finite-divided difference, (**ii**) backward finite-divided difference,	**20**	**CO4**

and (**iii**) centered finite-divided difference. Mention the **error** associated with each of the formula you employ.

x	0	2	4	6	8
***f*(*x*)**	1	7.3891	54.5982	403.4288	2980.9580

Q 12 (Marks: **20**; **CO5**)

Use the **Crank- Nicolson** method to solve for the temperature (T) distribution of a long thin rod with a length of 12 cm, at time (t) = 0.2 s. The following values: k = 0.85 cm^2/s, and ρ = 2.7 g/cm^3, step size (Δx) = 3 cm and time step (Δt) = 0.1 s. The equation is given below as:

$$k\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}$$

OR

Use Liebmann's method (Gauss-Seidel) to solve the steady-state heat equation without a heat source over the heated plate in **Fig. 2**. Employ over-relaxation with a value of **1.5** for the weighting factor and iterate to $\boldsymbol{\varepsilon_s = 25\%}$. The equation is given below as:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

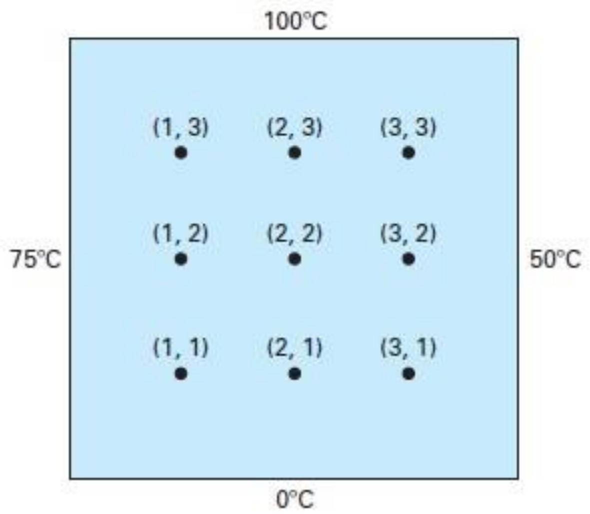


Fig 2: A heated plate where boundary temperatures are held at constant levels.

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CO5