Name:

Enrolment No:

UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2019

Course: Mathematical Physics-I (PHYS 1011) **Semester: I**

Programme: BSc Physics (H) **Time: 03 hrs.**

Max. Marks: 100

Number of pages: 03

Instructions:

SECTION A

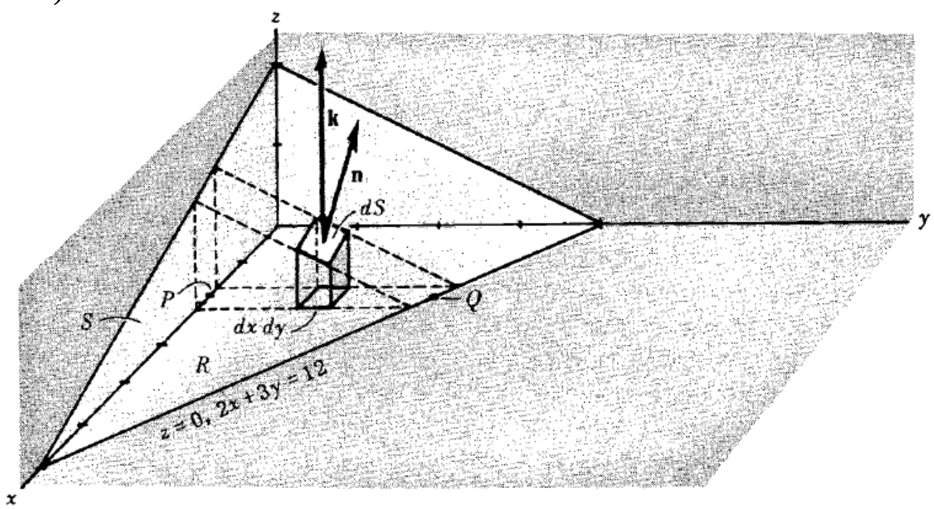
All questions are compulsory.

SN	Statement of Question	Marks	CO
Q1	Estimate the Wronskian corresponding to the following differential equation: $$(D^2 - 2D + 3)y = x^3 + \cos x$$ where $D = \frac{d}{dx}$	**4**	**CO1**
Q2	Find the directional derivative of the following scalar function $$f = x^2 - y^2 + 2z^2$$ at the point P (1,-2,-1) in the direction of the line PQ where Q is the point (5,0,4).	**4**	**CO3**
Q3	Define Dirac Delta function and state its properties.	**4**	**CO2**
Q4	Find $\vec{\nabla} \cdot \vec{F}$, where $\vec{F}$ is the gradient of the following scalar function $$u = x^3 + y^3 + z^3 - 3xyz$$	**4**	**CO3**
Q5	If $$\frac{d^2\vec{P}}{dt^2} = 6t\hat{\imath} - 12t\hat{\jmath} + 4\cos t\,\hat{k}$$ Find $\vec{P}$ if $\frac{d\vec{P}}{dt} = -\hat{\imath} - 3\hat{k}$ at $t = 0$ and $\vec{P} = 2\hat{\imath} + \hat{\jmath}$ at $t = 0$	**4**	**CO4**

SECTION B

Questions 6-8 are compulsory. There is an internal choice for the question number 9.

SN	Statement of Question	Marks	CO
Q6	Solve the following 2nd order linear differential equation with constant coefficient (find both general solution & particular integral): $$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = e^{-x}\sin x + xe^{3x}$$	**10**	**CO1**
Q7	a) Find the divergence and curl of the following vector field (in cylindrical coordinates): $$\vec{Q}(\rho, \varphi, z) = \rho\sin\varphi\,\hat{\rho} + \rho^2 z\hat{\varphi} + z\cos\varphi\,\hat{z}$$ where $\hat{\rho}, \hat{\varphi}$ and $\hat{z}$ are unit vectors in cylindrical coordinates. b) What is the physical significance of curl of a vector field? Evaluate curl of the following vector field $$\vec{A} = yz\,\hat{\imath} + 4xy\,\hat{\jmath} + y\hat{k}$$ at (1,-2,3).	**5+5**	**CO3**
Q8	Solve the following differential equation: $$\left(xy^2 - e^{1/x^3}\right)dx - x^2 y dy = 0$$	**10**	**CO2**

Q9	What do you mean by the flux of a vector field through a closed surface? Evaluate the flux of a vector field $\vec{A}$ through a surface S $$\iint_S \vec{A}\cdot\hat{n}\,ds$$ where $\vec{A} = 18z\hat{\imath} - 12\hat{\jmath} + 3y\hat{k}$, S is that part of the plane $2x + 3y + 6z = 12$ which is located in the first octant, and $\hat{n}$ is the unit normal to the surface S (see the figure below):  **OR**	**10**	**CO4**
Q9	State Stokes' theorem for a vector field by clearly defining each terms in the theorem. Evaluate the line integral of $\vec{F}$ ($\oint \vec{F}\cdot\overrightarrow{dr}$) on a closed surface C in the xy plane, $x = 2\cos t, y = 3\sin t$ from $t = 0$ to $t = 2\pi$.The vector field $\vec{F}$ is given as $$\vec{F} = (x - 3y)\hat{\imath} + (y - 2x)\hat{\jmath}$$	**10**	**CO4**
SECTION-C **Q10 is compulsory. There is an internal choice for Q11.**			
Q10	a) Define orthogonal curvilinear coordinate system. If (u_1, u_1, u_3) is a set of curvilinear coordinates, write an expression for the arc length in this coordinate system. b) When do we call a vector irrotational? Find the constants a, b, c so that the vector $$\vec{V} = (x + 2y + az)\hat{\imath} + (bx - 3y - z)\hat{\jmath} + (4x + cy + 2z)\hat{k}$$ is irrotational. c) Find the general solution of the following 1st order linear differential equation: $$x\log x\frac{dy}{dx} + y = \log x^2$$	**5+6+9**	**CO3+** **CO3+** **CO1**

Q11	a) Write the statement of Divergence theorem and discuss its physical significance. Using Divergence theorem, evaluate $$\iiint_V \vec{\nabla}\cdot\vec{F}\,dV$$ where $\vec{F} = 4xz\hat{\imath} - y^2\hat{\jmath} + yz\hat{k}$ and V is the volume of the cube bound by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. b) Verify Stokes' theorem for $$\vec{A} = (2x - y)\hat{\imath} - yz^2\hat{\jmath} - y^2z\hat{k}$$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary [Hint: Boundary C of S is a circle of radius one and center as origin in xy plane].	**10+10**	**CO4**
	OR		
Q11	a) Evaluate $$\iint_S (\vec{\nabla}\times\vec{A})\cdot\hat{n}\,ds$$ where $\vec{A} = (x^2 + y - 4)\hat{\imath} + 3xy\hat{\jmath} + (2xz + z^2)\hat{k}$ and S is the surface of a hemisphere $x^2 + y^2 + z^2 = 16$ above the xy plane. b) Find the work done in moving a particle in the force field $\vec{F} = 2x^3\hat{\imath} + (2z^2x - yz)\hat{\jmath} + xz\hat{k}$ along the curve defined by $x^2 = 4y, 3x^3 = 8z$ from $x = 0$ to $x = 2$.	**10+10**	**CO4**