Name: Enrolment No:	UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, December 2019

Programme Name: B. Sc. Mathematics (Hons.) **Semester : I**
Course Name : Calculus **Time : 03 hrs**
Course Code : MATH 1030 **Max. Marks : 100**
Nos. of page(s) : 02

Instructions: Attempt all questions from Section A. In Section B, Q6-Q8 are compulsory, and Q9 has internal choice. In Section C, Q10 is compulsory and Q11A and Q11B have internal choices.

SECTION A
(Attempt all questions)

S. No.		**Marks**	**CO**
Q1.	Using discriminant test examine whether the conic section $9x^2+6xy+y^2-12x-4y+4=0$ represents an ellipse, a parabola, or a hyperbola. Then show that the graph of the given quadratic equation is the line $y=-3x+2$.	**[4]**	**CO4**
Q2.	The x- and y-axes are rotated through an angle of $\frac{\pi}{2}$ radians about origin. Find an equation for the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, (a>b)$ in new coordinate.	**[4]**	**CO4**
Q3.	The position vector of a particle in space at time t is given by $\vec{r}(t)=(\sin t)\,\hat{\imath}+t\hat{\jmath}+(\cos t)\hat{k}$, $t\geq 0$. Find the time(s) when the velocity and acceleration vectors are orthogonal.	**[4]**	**CO4**
Q4.	If $I_n=\int_0^{\frac{\pi}{4}}\tan^n x\,dx$, then prove that a. $(n-1)(I_n+I_{n-2})=1$. b. $n(I_{n+1}+I_{n-1})=1$.	**[4]**	**CO2**
Q5.	Replace the polar equation $r=\frac{5}{\sin\theta-2\cos\theta}$ by equivalent Cartesian equation, and identify its graph.	**[4]**	**CO4**

SECTION B
(Q5-Q6 are compulsory and Q7 has internal choice)

S. No.		**Marks**	**CO**
Q6.	If $y=a\cos(\ln x)+b\sin(\ln x)$, prove that $$x^2y_{n+2}+(2n+1)xy_{n+1}+(n^2+1)y_n=0.$$	**[10]**	**CO1**
Q7.	Find the polar equation of the ellipse with eccentricity e and semi major axis a, considering one focus of the ellipse at origin and the corresponding directrix to the right of the origin. If $a=39$, $e=0.25$ find the distance from the focus to the associated directrix.	**[10]**	**CO4**

Q8.	Consider the vectors $\vec{a} = \hat{\imath} - \hat{\jmath} + \hat{k}$, $\vec{b} = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$ and $\vec{c} = -\hat{\imath} + 2\hat{\jmath} - \hat{k}$. Verify that $(\vec{a} \times \vec{b}).\vec{c} = (\vec{b} \times \vec{c}).\vec{a}$, and find the volume of the parallelepiped determined by $\vec{a}, \vec{b}$ and $\vec{c}$.	**[10]**	**CO4**
Q9.	Using first derivative test find two positive numbers whose sum is 20 and whose product is as large as possible. **OR** A rectangle is to be inscribed in a semicircle of radius 2. What is the largest area of the rectangle can have, and what are its dimensions?	**[10]**	**CO5**
	SECTION C **(Q10 is compulsory and Q11A and Q11B have internal choices)**		
Q10A.	State and prove Kepler's second law (the equal area law) of planetary motion.	**[10]**	**CO5**
Q10B.	Without finding $\vec{T}$ and $\vec{N}$, write the acceleration $\vec{a}$ of the motion $\vec{r}(t) = (2t+3)\hat{\imath} + (t^2 - 1)\hat{\jmath}$, in form of $\vec{a} = a_T\vec{T} + a_N\vec{N}$, where a_T and a_N are the tangential and normal components of acceleration, respectively.	**[10]**	**CO4**
Q11A.	Solve the following differential equation to find the position vector $\vec{r}(t)$ of a particle. $$\frac{d^2\vec{r}}{dt^2} = -(\hat{\imath} + \hat{\jmath} + \hat{k}),$$ with initial conditions $\vec{r}(0) = 10\hat{\imath} + 10\hat{\jmath} + 10\hat{k}$, and $\left.\frac{d\vec{r}}{dt}\right\rvert_{t=0} = \vec{0}$. **OR** Suppose that $\vec{r_1}(t) = f_1(t)\hat{\imath} + f_2(t)\hat{\jmath} + f_3(t)\hat{k}$, $\vec{r_2}(t) = g_1(t)\hat{\imath} + g_2(t)\hat{\jmath} + g_3(t)\hat{k}$, $\lim_{t \to t_0} \vec{r_1}(t) = \vec{A}$ and $\lim_{t \to t_0} \vec{r_2}(t) = \vec{B}$. Use the determinant formula for cross products and the limit product rule for scalar functions to show that $$\lim_{t \to t_0} (\vec{r_1}(t) \times \vec{r_2}(t)) = \vec{A} \times \vec{B}.$$	**[10]**	**CO4**
Q11B.	Find the volume of the solid generated by revolving the region bounded by the curves $y = x^2, y = 0$ and $x = 2$ about x-axis. **OR** Find the area of the surface generated by revolving the curve $x = \frac{y^3}{3}, 0 \leq y \leq 1$ about y-axis.	**[10]**	**CO3**

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