| Name: <br> Enrolment No: | UPES — UNIVERSITY WITH A PURPOSE |
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**UNIVERSITY OF PETROLEUM AND ENERGY STUDIES**
**End Semester Examination, December 2019**

**Course: Algebra** — **Semester: I**
**Program: B.Sc. (Hons.) Mathematics** — **Time : 03 hrs.**
**Course Code: MATH 1032** — **Max. Marks: 100**

**Instructions: All questions are compulsory.**

**SECTION A**

| S. No. | | Marks | CO |
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| Q1 | Represent the complex number $z = \left(1-\sqrt{3}i\right)^3$ in polar coordinates $r$ and $\theta$. | **4** | **CO1** |
| Q2 | Prove or disapprove the statement : <br> *An odd degree polynomial $f:\mathbb{R}\longrightarrow\mathbb{R}$ is always an onto function* | **4** | **CO2** |
| Q3 | Show that the set of reals $\mathbb{R}$ is cardinally equivalent to a subset $(0,1)$ of it. | **4** | **CO2** |
| Q4 | Suppose $\omega$ and $\omega^2$ are the cube roots of unity other than 1. Find the trace of the matrix $\begin{pmatrix}1 & 3 & 2\\ 0 & \omega & 3\\ 0 & 0 & \omega^2\end{pmatrix}^{2019}$ | **4** | **CO4** |
| Q5 | Let $M_{2\times2}(\mathbb{R})$ be the vector space of all $2\times2$ real matrices. Consider the subspaces <br> $W_1 = \left\{\begin{pmatrix}a & -a\\ c & d\end{pmatrix} : a,c,d\in\mathbb{R}\right\}$ and $W_2 = \left\{\begin{pmatrix}a & b\\ -a & d\end{pmatrix} : a,b,d\in\mathbb{R}\right\}$ <br> Find the dimensions of subspaces $W_1\cap W_2$ and $W_1+W_2$ respectively. | **4** | **CO5** |

**SECTION B**

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| Q6 | Prove that $z = x+iy, i=\sqrt{-1}$ is either real or purely imaginary if and only if $(\overline{z})^2 = z^2$. | **10** | **CO1** |
| Q7 | Consider the set $A = \{1,2,3,\ldots\ldots,9,10\}$ and $\approx$ be the relation on $A\times A$ defined by <br> $(a,b)\approx(c,d)$ whenever $ad=bc$ <br> Prove that $\approx$ is an equivalence relation. Find $[(2,4)]$ i.e. the equivalence class of $(2,4)$. | **10** | **CO2** |
| Q8 | Use the principle of mathematical induction to prove: <br> $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\ \sin n\theta\ , n\in\mathbb{N}$ | **10** | **CO3** |

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| Q9 | Prove that<br>$\bigcap_{i=1}^{n} W_i$<br>is a subspace of a vector space $V$ over the field $F$,where $W_i, 1 \leq i \leq n$ are subspaces of $V(F)$. | **10** | **CO3** |
| | **SECTION-C** | | |
| Q 10 | a. Suppose $\mathcal{N}(A)$ denotes the dimension of the null space of matrix<br>$A = \begin{pmatrix} 2 & 2 & -6 & 8 \\ 3 & 3 & -9 & 8 \\ 1 & 1 & x & 4 \end{pmatrix}.$<br>For what values of $x$, $\mathcal{N}(A)$ is minimum?<br><br>b. Consider the following subspace of $\mathbb{R}^3$:<br>$W = \{(x, y, z) \in \mathbb{R}^3 \mid 2x + 2y + z = 0, 3x + 3y - 2z = 0, x + y - 3z = 0\}$<br>Find a basis and the dimension of $W$. | **10<br>+<br>10** | **CO4** |
| Q11 | Let $V$ be the real vector space of all polynomials from $\mathbb{R}$ into $\mathbb{R}$ of degree 2 or less. Let $t$ be a fixed real number and define $g_1(x) = 1, g_2(x) = (x + t), g_3(x) = (x + t)^2$ such that $\mathcal{B} = \{g_1, g_2, g_3\}$ is a basis for $V$. Find $[f(x)]_{\mathcal{B}}$ i.e. the coordinates of $f(x) = c_0 + c_1 x + c_2 x^2$ in this ordered basis $\mathcal{B}$.<br><br>OR<br><br>Let $V$ be the set of $2 \times 2$ matrices $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ with complex entries such that $a_{11} + a_{22} = 0$. Let $W$ be the set of matrices in $V$ with $a_{12} + \overline{a_{21}} = 0$. Prove that :<br><br>a. $V$ is a vector space over $\mathbb{C}$.<br>b. $W$ is a vector space over $\mathbb{R}$.<br>c. Is $W$ is a vector space over $\mathbb{C}$ ? Give reason to support your answer. | **20** | **CO5** |