Name: **Enrolment No:**	UPES UNIVERSITY WITH A PURPOSE

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES
End Semester Examination, May 2019

Course: B.Sc. **Semester: II**
Program: Mathematics Hons. **Time 03 hrs.**
Course Code: MATH1018 **Max. Marks: 100**

Instructions:

SECTION A
(Attempt all questions)

S. No.		Marks	CO
Q 1	Write the following numbers in ternary and hence identify whether they are the elements of Cantor's set or not a. $\frac{5}{9}$ b. $\frac{3}{10}$	**4**	**CO1**
Q 2	Give an example of a set which is not dense but fails to be nowhere dense.	**4**	**CO1**
Q 3	Find limit superior and limit inferior of the following sequences (i) $\left\{\cos\left(\pi+\frac{1}{n}\right)\right\}$ (ii) $\left\{\sin\left(\frac{(-1)^n\pi}{2}+\frac{1}{n}\right)\right\}$	**4**	**CO2**
Q 4	Find all cluster points of the sequence $\left\{\frac{1}{\sqrt{n}}\left(\frac{1}{\sqrt{1}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{5}}+\ldots+\frac{1}{\sqrt{2n-1}+\sqrt{2n+1}}\right)\right\}$	**4**	**CO2**
Q 5	Show that the sum of infinite series $\sum_{n=1}^{\infty}\frac{1}{n(n+1)}=1$	**4**	**CO3**
SECTION B			
(Q6-Q8 are compulsory and Q9 has internal choice)			
Q 6	Prove that every subset of countable set is countable.	**10**	**CO1**
Q 7	If $\{a_n\}$ is a sequence of real numbers such that $0<a_1<a_2$ and $a_n=\frac{2a_{n-1}a_{n-2}}{a_{n-1}+a_{n-2}}$, then show that $\lim_{n\to\infty}a_n=\frac{3a_1a_2}{2a_1+a_2}$.	**10**	**CO2**
Q 8	Let $\{a_k\}$ be an unbounded strictly increasing sequence of positive real numbers and	**10**	**CO2**

	$x_k = \frac{a_{k+1} - a_k}{a_{k+1}}$. Then prove that for all $n \ge m$, $\sum_{k=m}^{n} x_k > 1 - \frac{a_m}{a_n}$.		
Q 9	Discuss the convergence of the series $1 + \frac{1}{2}\frac{x^2}{4} + \frac{1.3.5}{2.4.6}\frac{x^4}{8} + \frac{1.3.5.7.9}{2.4.6.8.10}\frac{x^6}{12} + \ldots$ **OR** If $\sum a_n$ is a convergent series of real numbers then show that $\sum b_n = a_{n+1} - a_n$ (telescopic series) is also a convergent series.	**10**	**CO3**
SECTION-C **(Q10 is compulsory and Q11 has internal choice)**			
Q 10	a. If $\sum S_n = 1 - \frac{1}{2^p} + \frac{1}{3^p} - \frac{1}{4^p} + \ldots, p > 0$, then find the conditions on p for (i) Absolute convergence of $\sum S_n$ (ii) Conditional convergence of $\sum S_n$ b. Prove that there are countably infinite end points of all removed open intervals in Cantor's set.	**10+10**	**CO3** **CO1**
Q 11	a. If $\{a_n\}$ and $\{b_n\}$ are two sequences of positive real numbers such that (i) $\frac{2}{a_{n+1}} = \frac{1}{a_n} + \frac{1}{b_n}$ (ii) $b_{n+1} = \frac{a_n + b_n}{2}$ If a_1 and b_1 are given, then show that both sequences are convergent and converge to the same limit. b. If $Y = \left\{ \frac{x}{1+\|x\|}, x \in R \right\}$, then find the set of all limit points of Y. **OR**	**10+10**	**CO2** **CO1**
Q 11	a. Let the sequence $\{a_n\}$ defined by $a_{n+1} = \frac{1}{2}\left(a_n + \frac{2019^2}{a_n}\right)$ such that $a_1 > 0$. Prove the following statements (i) $\{a_n\}$ is monotonic (ii) $\{a_n\}$ is bounded (iii) $\lim_{n \to \infty} a_n = 2019$ b. Let $S = ¿ n = 1 ¿ \infty \left(\left[0, \frac{1}{2n+1}\right] \cup \left[\frac{1}{2n}, 1\right] \right)$. Then show that $[0,1]$¿ is an open set.	**10+10**	**CO2** **CO1**

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Instructions:

SECTION A
(Attempt all questions)

S. No.		**Marks**	**CO**
Q 1	Write the following numbers in ternary and hence identify whether they are the elements of Cantor's set or not a. $\frac{1}{4}$ b. $\frac{1}{3}$	**4**	**CO1**
Q 2	Give an example of a set which is nowhere dense.	**4**	**CO1**
Q 3	Find limit superior and limit inferior of the following sequences (iii) $\left\{\sin\left(\pi+\frac{1}{n}\right)\right\}$ (iv) $\left\{\tan\left(\frac{(-1)^n\pi}{2}+\frac{1}{n}\right)\right\}$	**4**	**CO2**

Q 4	Find all cluster points of the sequence $\left\{\left(\frac{1}{(n+1)^2}+\frac{1}{(n+2)^2}+\ldots+\frac{1}{(2n)^2}\right)\right\}$	**4**	**CO2**
Q 5	Show that the sum of infinite series $\frac{1}{1!}+\frac{1+2}{2!}+\frac{1+2+3}{3!}+\ldots=\frac{3e}{2}$	**4**	**CO3**
SECTION B **(Q6-Q8 are compulsory and Q9 has internal choice)**			
Q 6	Let $X=(0,1)(2,3)$ be an open set in R. Let f be a continuous function on X such that the derivative $f'(x)=0$ for all x. Then accept and reject the following statements with proper argument. a. Then the range of f has uncountable number of points b. Countably infinite number of points c. At most 2 points d. At most 1 point	**10**	**CO1**
Q 7	If $\{a_n\}$ is a sequence of real numbers such that $a_{n+1}=\sqrt{5+a_n}, a_1=0$, then prove that a_n in monotonic and bounded. Also, find the unique limit point of $\{a_n\}$.	**10**	**CO2**
Q 8	Let $\{a_k\}$ be an unbounded strictly increasing sequence of positive real numbers and $x_k=\frac{a_{k+1}-a_k}{a_{k+1}}$. Then prove that for all $n\geq m$, $\sum_{k=m}^{n} x_k>1-\frac{a_m}{a_n}$.	**10**	**CO2**
Q 9	Discuss the convergence of the series $1+\frac{1}{2.4}+\frac{1.3.5}{2.4.6.8}+\frac{1.3.5.7.9}{2.4.6.8.10.12}+\ldots$ **OR** If $\sum a_n^2$ is a convergent series of real numbers then show that $\sum \frac{a_n}{n}$ is also a convergent series.	**10**	**CO3**
SECTION-C **(Q10 is compulsory and Q11 has internal choice)**			
Q 10	(i) Test the series $\sum S_n=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\ldots$ then find the conditions on x for absolute convergence of series. (ii) Show that the collection of all sequences of 0s and 1s is uncountable and equivalent to P(N¿.	**10+10**	**CO3** **CO1**
Q 11	(a) If $\{a_n\}$ and $\{b_n\}$ are two sequences of positive real numbers such that (i) $a_{n+1}=\sqrt{a_n b_n}$ (ii) $b_{n+1}=\frac{a_n+b_n}{2}$ If a_1 and b_1 are given, then show that both sequences are convergent and they	**10+10**	**CO2** **CO1**

	converge to the same limit. (b) Using transfinite numbers show that line, plane and space are similar and have cardinality equal to the cardinality of continuum. **OR**		
Q 11	(a) Let the sequence $\{a_n\}$ defined by $a_{n+1}=\frac{1}{2}\left(a_n+\frac{9}{a_n}\right)$ such that $a_1>0$. Prove the following statements (i) $\{a_n\}$ is monotonic (ii) $\{a_n\}$ is bounded (iii) $\lim_{n \to \infty} a_n = 3$ (b) Let $A=\{1,2,\ldots,10\}$. If S is a subset of A, and let $¿S \vee ¿$ denotes the number of elements in S. Then find $\sum_{S \subset A, S \neq \phi} (-1)^{¿S \vee ¿} ¿$.	**10+10**	**CO2** **CO1**