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UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, May 2018

Programme: B. Tech. (EE, BCT)	**Semester – IV**
Course Name: Probability and Random Variables	**Max. Marks : 100**
Course Code: MATH 221	**Duration : 3 Hrs**
No. of page/s:	

Instructions:
Attempt all questions from **Section A** (each carrying 4 marks); attempt all questions from **Section B** (each carrying 10 marks)**;** attempt all questions from **Section C** (each carrying 20 marks)**.**

	Section A **(Attempt all questions)**		
1.	There are 3 true coins and 1 false coin with head on both sides. A coin is chosen at random and tossed 4 times. If head occurs all the 4 times, what is the probability that the false coin has been chosen and used?	**[4]**	**CO1**
2.	Define power spectral density function and show that the spectral density function of a real random process is an even function.	**[4]**	**CO3**
3.	Find the characteristic function of the Laplace distribution with probability density function $f(x)=\frac{\alpha}{2}e^{-\alpha\lvert x\rvert},\ -\infty<x<\infty$.	**[4]**	**CO2**
4.	Show that the sum of two independent Poisson processes is a Poisson process.	**[4]**	**CO4**
5.	Describe the Bernoulli process and construct a typical sample sequence of the Bernoulli process.	**[4]**	**CO3**
	SECTION B **(Q6-Q8 are compulsory and Q9 has internal choice)**		
6.	Assume that the probability of an individual coalminer being killed in a mine accident during a year is 1/2400. Use Poisson's distribution to calculate the probability that in a mine employing 200 miners there will be at least one fatal accident in a year.	**[10]**	**CO1**

7.	The joint probability density function of a two dimensional random variable (X,Y) is given by $f(x,y)=xy^2+\frac{x^2}{8}, \quad 0\le x\le 2,\ 0\le y\le 1.$ Compute $P(X>1), P\left(Y<\frac{1}{2}\right), P\left(X>1 \big/ Y<\frac{1}{2}\right), P(X<Y)$ and $P(X+Y\le 1)$.	**[10]**	**CO2**
8.	Consider a two-state Markov chain with the transition probability matrix $$P=\begin{bmatrix}1-a & a\\ b & 1-b\end{bmatrix} \qquad 0<a<1, 0<b<1$$ Assume that $a=0.1$ and $b=0.2$, and the initial distribution is $P(X_0=0)=P(X_0=1)=0.5$. i) Find the distribution of X_n . ii) Find the distribution of X_n when $n\to\infty$.	**[10]**	**CO4**
9.	The process $\{X(t)\}$ whose probability distribution under certain conditions is given by $$P\{X(t)=n\}=\frac{(at)^{n-1}}{(1+at)^{n+1}}, \quad n=1,2,...$$ $$=\frac{at}{1+at}, \qquad n=0.$$ Show that it is not stationary. **OR** If the power spectral density of a WSS process is given by $$s(\omega)=\begin{cases}\frac{b}{a}(a-\lvert\omega\rvert), & \lvert\omega\rvert\le a\\ 0, & \lvert\omega\rvert>a\end{cases}.$$ Find the autocorrelation function of the process.	**[10]**	**CO3**

SECTION C
(Q10 is compulsory and Q11 has internal choice)

10.	**a)** In the fair coin experiment, we define the process $\{X(t)\}$ as follows. $$X(t) = \sin \pi t, \quad \text{if head shows, and} \\ = 2t, \quad \text{if tail shows.}$$ i) Find $E\{X(t)\}$ and ii) Find $F(x,t)$ for $t = 0.25$.	**[10]**	**CO3**
	b) If the probability density of X is given by $$f(x) = \begin{cases} \frac{1}{\theta} e^{-\frac{x}{\theta}}, & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}$$ Find the mean and variance of X .	**[10]**	**CO1**
11.	If people arrive to purchase cinema tickets at the average rate of 6 per minute, it takes an average of 7.5 seconds to purchase a ticket. If a person arrives 2 min before the picture starts and it takes exactly 1.5 min. to reach the correct seat after purchasing the ticket, i) Can he expect to be seated for the start of the picture? ii) What is the probability that he will be seated for the start of the picture? iii) How early he arrive in order to be 99% sure of being seated for the start of the picture? **OR** Customers arrive at a one man barber shop according to a Poisson process with a mean interarrival time of 12 min. Customers spend an average of 10 min in the barber's chair. i) What is the expected number of customers in the barber shop and in the queue? ii) Calculate the percentage of time an arrival can walk straight into the barber's chair without having to wait. iii) How much time can a customer expect to spend in the barber's shop? iv) What is the average time customers spend in the queue? v) What is the probability that the waiting time in the system is greater than 30 min?	**[20]**	**CO4**