Roll No: ______________________________

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, May 2018

Programme: B.Tech (All SOCS Programs) | **Semester – II**
Course Name: Mathematics II | **Max. Marks : 100**
Course Code: MATH 1005 | **Duration : 3 Hrs**
No. of page/s:3

Section A
(Attempt all questions)

		MARKS	
1.	If the mean of the Poisson distribution is 2, find $P(r \geq 1)$.	[4]	CO2
2.	Let $A = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24\}$ be ordered by the relation R defined by '$xRy \Leftrightarrow x\ divides\ y$'. Draw the Hasse diagram of the poset (A, R).	[4]	CO5
3.	Can $\sqrt{2}$ be approximated through fixed-point iteration formula $x_{n+1} = \emptyset(x)$? If so, find the function $\emptyset(x)$.	[4]	CO3
4.	The first four moments of a distribution about the value 4 of the variable are $-1.5, 17, -30$ and 108. State whether the distribution is leptokurtic or platykurtic.	[4]	CO2
5.	Solve the initial value problem $4y'' - y = 0, y(0) = 2, y'(0) = \beta$. Then find β so that the solution approaches zero as $t \to \infty$.	[4]	CO1

SECTION B
(All questions are compulsory, Q10 has internal choice)

6. The following data was collected when a large oil tanker was loading: **[08]** **CO4**

t, *min*	0	15	30	45	60
$V, 10^6$barrels	0.5	0.65	0.73	0.88	1.03

Calculate the flow rate $(i.e., \frac{dV}{dt})$ at $t = 5$.

7. Solve $x^3 \frac{d^3y}{dx^3} + 3x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = x + \log x$. **[08]** **CO1**

8. Consider the poset $(\{ \{1\}, \{2\}, \{4\}, \{1,2\}, \{1,4\}, \{2,4\}, \{3,4\}, \{1,3,4\}, \{2,3,4\} \}, \subseteq)$. **[08]** **CO5**

(a) Find the maximal elements
(b) Find the minimal elements
(c) Find all the upper bounds of $\{\{2\}, \{4\}\}$ and the least upper bound, if it exists.
(d) Find all the lower bounds of $\{1,3,4\}$ and the greatest lower bound, if it exists.

9.	The following system of equations is designed to determine concentrations in a series of coupled reactors as a function of the amount of mass input to each reactor: $$-3c_1 + 18c_2 - 6c_3 = 1200$$ $$15c_1 - 3c_2 - c_3 = 3800$$ $$-4c_1 - c_2 + 12c_3 = 2350$$ Obtain the concentration values correct to 2 decimals by using *Gauss-Seidel* iterative technique with initial approximate solution as $\left[c_1^{(0)}, c_2^{(0)}, c_3^{(0)}\right] = [300, 220, 310]$.	**[08]**	**CO3**
10.	The following table gives the values of the probability integral $f(x) = \frac{1}{\sqrt{2\pi}}\int_0^x e^{-x^2}\,dx$ for certain equidistant values of x. Find the value of $\frac{1}{\sqrt{2\pi}}\int_0^{0.543} e^{-x^2}\,dx$ using the data given below: <table><tr><td>x</td><td>0.51</td><td>0.52</td><td>0.53</td><td>0.54</td><td>0.55</td></tr><tr><td>$f(x)$</td><td>0.5292</td><td>0.5379</td><td>0.5465</td><td>0.5549</td><td>0.5633</td></tr></table> **(OR)** A reservoir discharging water through sluices as a depth h meter below the water surface, has a surface area A for various values of h as given below: <table><tr><td>$h(meter)$</td><td>10</td><td>11</td><td>12</td><td>13</td><td>14</td></tr><tr><td>$A\ (sq.meter)$</td><td>950</td><td>1070</td><td>1200</td><td>1350</td><td>1530</td></tr></table> If t denotes time in minutes, the rate of fall of surface area is given by $\frac{dh}{dt} = \frac{-48\sqrt{h}}{A}$. Using Simpson's 1/3rd rule, estimate the time taken for the water level to fall from 14 to 10 meters above the sluices.	**[08]**	**CO4**

SECTION C

(Q11 is compulsory and Q12A, Q12B have internal choice)

11.A	Solve $\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + (4x^2 - 1)y = -3e^{x^2}\sin 2x$ by the removal of the first derivative (reducing it into normal form) method.	**[10]**	**CO1**
11.B	Consider the following table <table><tr><td>x</td><td>1</td><td>3</td><td>4</td><td>5</td></tr><tr><td>*f(x)*</td><td>0</td><td>1.0986</td><td>1.3865</td><td>1.6094</td></tr></table> Construct the interpolating polynomial using Newton's divided difference interpolation formula and hence obtain the values of $f(1.5)$ and $f(4.5)$.	**[10]**	**CO4**

12.A	In a sample of 1000 cases, the mean of a certain test is 14 and S.D is 2.5. Assuming the distribution to be normal, find (i) How many students score between 12 and 15? (ii) How many score above 18? Given that the area under the standard normal curve between $z = 0$ and $z = 0.8$ is 0.2881, between $z = 0$ and $z = 0.4$ is 0.1554 and between $z = 0$ and $z = 1.6$ is 0.4452 **(OR)** If 10% of the bolts produced by a machine are defective, determine the probability that out of 10 bolts chosen at random (i) 1 is defective (ii) at most 2 will be defective.	**[10]**	**CO2**
12.B	Consider the initial value problem $$\frac{dy}{dx} = xy^{\frac{1}{3}},\ y(1) = 1.$$ Using step size $h = 0.1$, Find the value of $y(1.1)$ by (i) Taylor series method (considering derivatives up to third order) (ii) Modified Euler's method. **(OR)** Solve the following initial value problem to obtain $y(1)$ using fourth order Runge-Kutta method with step size $h = 0.5$. $$\frac{dy}{dx} = yx^2 - 1.2y, \qquad y(0) = 1$$	**[10]**	**CO3**

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Section A
(Attempt all questions)

1.	Probability density function of a continuous random variable X is given by $f(x) = ke^{-x}, 0 \leq x < \infty$. Find k and hence evaluate $P(0 < X < 1)$.	**[4]**	**CO2**
2.	Show that $(\mathbb{Z}^+, /)$ is a poset, where a/b is defined as $'a\ divides\ b'$.	**[4]**	**CO5**
3.	Show that iterative formula $x_{n+1} = \frac{1}{k}\left[(k-1)x_n + \frac{N}{x_n^{k-1}}\right]$ to find $\sqrt[k]{N}$ using Newton's Raphson method.	**[4]**	**CO3**
4.	In a binomial distribution consisting of 5 independent trials, probabilities of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the parameter p of the distribution and hence find $P(X > 1)$.	**[4]**	**CO2**
5.	Solve $xy^2dx - x^2ydy - e^{\frac{1}{x^3}}dx = 0$	**[4]**	**CO1**

SECTION B
(Q6-Q9 are compulsory and Q10 has internal choice)

6. From the following table a half-yearly premium for policies matching at different ages, estimate the premium for a policy matching at the age of 46 and 49. **[8]** **CO4**

Age	45	50	55	60	65
Premium ($)	114.84	96.16	83.32	74.48	68.48

7.	Solve $\left(\frac{d^2y}{dx^2} + y\right)cotx + 2\left(\frac{dy}{dx} + y\ tanx\right) = secx.$	**[8]**	**CO1**
8.	Let $A = \{2, 3, 4, 6, 12, 36, 48\}$ be a non-empty set and R be the partial order relation of divisibility defined on A. That is, if $a, b \in A$, then a divides b. Draw Hasse diagram of R.	**[8]**	**CO5**

<table>
<tr><td>9.</td><td>If p, q, r, s are the successive entries corresponding to equidistant arguments in a table, show that when the third differences are taken into account, the entry corresponding to the argument half way between the arguments at q and r is $\left[A+(B/24)\right]$, where A is the arithmetic mean of q and r and B is arithmetic mean of $3q-2p-s$ and $3r-2s-p$.</td><td>[8]</td><td>CO4</td></tr>
<tr><td>10.</td><td>Using appropriate Simpson's rule, evaluate $\int_0^{\frac{\pi}{2}} \sqrt{\cos\theta}\, d\theta$, taking 10 ordinates.

OR

The voltage $E = E(t)$ in an electrical circuit obeys the equation $E = L\frac{dI}{dt} + R\, I(t)$, where R is the resistance and L is the inductance. Use $L = 0.05$ and $R = 2$ and values for $I(t)$ in the table given below, find $E(1.4)$

<table>
<tr><td>t</td><td>1</td><td>1.1</td><td>1.2</td><td>1.3</td><td>1.4</td></tr>
<tr><td>$I(t)$</td><td>8.2277</td><td>7.2428</td><td>5.9908</td><td>4.5260</td><td>2.9122</td></tr>
</table>
</td><td>[8]</td><td>CO4</td></tr>
<tr><td colspan="4">SECTION C
(Q11 is compulsory and Q12 has internal choice)</td></tr>
<tr><td>11A.</td><td>Find the solution of differential equation $y_2 + (1 - cotx)y_1 - cotx\, y = \sin^2 x$.</td><td>[10]</td><td>CO1</td></tr>
<tr><td>11B.</td><td>Solve the given system of equations
$27x + 6y - z = 85$; $6x + 15y + 2z = 72$; $x + y + 54z = 110$ by Gauss-Seidel method correct to three decimal places.</td><td>[10]</td><td>CO3</td></tr>
<tr><td>12 A.</td><td>A certain number of articles manufactured in one batch were classified into three categories according to a particular characteristic, being less than 50, between 50 and 60 and greater than 60. If this characteristic is known to be normally distributed, determine the mean and variation for this batch if 60% 35% and 5% were found in these categories. $P[0 < z < 0.2533] = 0.1$; $P[0 < z < 1.645] = 0.45$.

OR

In sampling a large number of parts manufactured be a machine, then mean number of defectives in a sample of 20 is 2. Out of 1000 such samples, how many would be expected to contain (i) Exactly 3 defective parts (ii) At most 3 defective parts (iii) At least 3 defective parts.</td><td>[10]</td><td>CO2</td></tr>
<tr><td>12B.</td><td>Find a positive real root of given equation $2x - \log_{10} x = 7$ using fixed-point iteration method correct to four decimal places.
OR

Using Runge-Kutta method of fourth order, find y for $x = 0.1, 0.2$ given that
$\frac{dy}{dx} = xy + y^2$, $y(0) = 1$ and step size $h = 0.1$.</td><td>[10]</td><td>CO3</td></tr>
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