UPES

UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, May 2018

Programme: B.Tech. (APE-UP, FSE) **Semester – IV**
Course Name: Applied Numerical Methods **Max. Marks : 100**
Course Code: MATH-307 **Duration : 3 Hrs**
No. of page/s: 02

Instructions:
Attempt all questions from **Section A** (each carrying 5 marks); attempt all questions from **Section B** (each carrying 8 marks); attempt all questions from **Section C** (each carrying 20 marks).

SECTION A **(Attempt all questions)**			
1.	Round off the number 299.995 to 2 decimal places and compute the relative error in your answer.	**[5]**	**CO1**
2.	Consider the equation $f(x)=x^2-x-1$. In order to obtain the zero of $f(x)$ in the interval $(1,2)$, show that the fixed-point iteration scheme $$x=\left(\frac{x^2+1}{2x-1}\right)=\varphi(x)$$ has a second order convergence.	**[5]**	**CO1**
3.	Suppose k is real and $f(x)=kx^4+1$. If the fourth order divided difference of $f(x)$ at the points $1,2,3,4,5$ is 5 then find the value of k.	**[5]**	**CO2**
4.	Use composite Trapezoidal rule to evaluate $\int_0^1\int_0^1 dxdy$ by dividing the range of integration into two equal parts.	**[5]**	**CO3**
SECTION B **(Q5-Q8 are compulsory and Q9 has internal choice)**			
5.	Suppose $p(x)$ is a polynomial of degree 2 that interpolates the data $(-1,2),(0,1)\wedge(1,2)$. If $q(x)$ is a polynomial of degree 3 such that $p(x)+q(x)$ interpolates the data $(-1,2),(0,1),(1,2)$ and $(2,11)$, then find $q(3)$.	**[8]**	**CO2**
6.	Compute the definite integral $$I=\int_{-2}^{2} max\{\lvert x^3\rvert,x^2\}dx$$ using Simpson's rule by dividing the interval $[-2,2]$ into 4 equal parts. Also compare the result with the actual value of the integral and calculate the absolute error in the calculated value of I.	**[8]**	**CO3**
7.	Use Taylor's series method to obtain $y(0.1)$ correct to 3decimal places, if given that $\frac{dy}{dx}=y+x, y(0)=1$.	**[8]**	**CO5**

8.	The fourth order Runge−¿Kutta method $u_{j+1}=u_j+\frac{1}{6}\left[K_1+2K_2+2K_3+K_4\right]$ is used to solve the initial value problem: $\frac{du}{dt}=u, u(0)=\alpha.$ If $u(1)=1$ is obtained by taking the step size $h=1$,then find the value of α .	**[8]**	**CO5**
9.	Solve $u_t=5u_{xx}$ with $u(0,t)=0; u(5,t)=60$ and $u(x,0)=\begin{cases} 20x \; for \; 0<x\leq 3 \\ 60 \; for \; 3<x\leq 5 \end{cases}$; for five time steps taking $h=1$ by using Bender-Schmidt method. **OR** Solve $u_t=u_{xx}$ with $u(x,0)=0; u(0,t)=0$ and $u(1,t)=1$. Compute u for $t=1/8$ in two time steps, using Crank-Nicholson's method.	**[8]**	**CO6**
SECTION C **(Q10 has internal choice and Q11 is compulsory)**			
10.	Suppose $k$is non-prime and the matrix $A=\begin{bmatrix} 1 & 1 & k \\ 2 & k & 2 \\ 1 & 3 & 2 \end{bmatrix}$ is such that $det(A)=-1$. Consider the unique decomposition $A=LU$, where $L=\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}$ and $U=\begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$. Let $X\in R^3 \wedge b=[1,1,1]^t$. Find the solution of the system $Ax=b$ where $x=[x,y,z]^t$. **OR** Suppose $k$is positive and the matrix $A=\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & k \\ 1 & k & 3 \end{bmatrix}$ is such that $det(A)=1$. Consider the unique decomposition $A=LU$, where $L=\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}$ and $U=L^T$, where L^T denotes the transpose matrix of L. Let $X\in R^3 \wedge b=[1,1,3]^t$. Find the solution of the system $Ax=b$ where $x=[x,y,z]^t$.	**[20]**	**CO4**
11.	Consider an IVP:	**[20]**	**CO5**

	$$\frac{dy}{dx} = \lvert x-1 \rvert + y, \; y(0) = 1$$ Find the value of $y(1)$ using Euler's method with $h = \frac{1}{4}$. Also obtain the actual solution of the given IVP and compute the absolute error in the calculated value.		