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UNIVERSITY OF PETROLEUM AND ENERGY STUDIES

End Semester Examination, May 2018

Programme:	**B.Tech (Mechanical, ADE)**	**Semester – VI**	
Course Name:	**Applied Numerical Techniques**	**Max. Marks**	**: 100**
Course Code:	**MATH 305**	**Duration**	**: 03 Hrs.**
No. of page/s:	**03**		

Instructions:

Attempt all questions from **Section A** (each carrying 5 marks)**;** attempt all questions from **Section B** (each carrying 8 marks)**;** attempt all questions from **Section C** (each carrying 20 marks)**.**

Section A
(Attempt all questions)

1.	Express the function $f(x) = 2x^3 - 3x^2 + 3x - 10$ in factorial notation.	**[5]**	**CO1**
2.	Let $x_0 = 1.5$ be the initial approximation of a root of the equation $$x^2 + \log_e x - 2 = 0.$$ Find an approximate root of the equation using fixed point iteration method (iteration method), correct upto three significant digits.	**[5]**	**CO3**
3.	By considering five terms of Taylor's series, evaluate $y(0.2)$ from the following differential equation $$\frac{dy}{dx} = 2y + 3e^x, y(0) = 0.$$	**[5]**	**CO5**
4.	Consider the following boundary value problem (BVP). $$\frac{d^2y}{dx^2} - y = x^4, 0 \le x \le 1,$$ with the boundary conditions $y(0) = 0$ and $y(1) = 0$. Choose two basis functions $\phi_1(x)$ and $\phi_2(x)$ for an approximate solution $\bar{y} = a_1\phi_1(x) + a_2\phi_2(x)$. Hence find the residual.	**[5]**	**CO6**

SECTION B
(Q5-Q8 are compulsory and Q9 has internal choice)

5.	Solve the equation $x - \log_e(2x + 2) = 0$, for the root in (1,2), by Regula-Falsi method, correct upto four significant digits.	**[8]**	**CO3**

6.	Use fourth order Runge-Kutta method to solve for $y(1.2)$, considering step-length $h = 0.1$, given that $$\frac{dy}{dx} = x^2 + y^2,$$ with initial condition $y(1) = 1.5$.	**[8]**	**CO5**
7.	Evaluate the integration $\int_0^{\frac{\pi}{2}} \sqrt{1 - k\sin^2\phi}\, d\phi, k = 0.162$, by Simpson's $\frac{1}{3}$ rule, dividing the interval $0 \le \phi \le \frac{\pi}{2}$ into six equal subintervals.	**[8]**	**CO2**
8.	Intensity of radiation is directly proportional to the amount of remaining radioactive substance. The differential equation is $$\frac{dy}{dx} = -ky, \text{ where } k = 0.01.$$ Given that $x_0 = 0$ and $y_0 = 100$. Determine how much substance will remain at the moment $x = 100$, using Modified Euler's method with the step-length $h = 50$.	**[8]**	**CO5**
9.	Interchange the equations of the following system to obtain a strictly diagonally dominant system. Then apply Gauss-Seidel method to evaluate an approximate solution, taking the initial approximation as $x_1^{(0)} = 1, x_2^{(0)} = 1, x_3^{(0)} = 1$. Perform three iterations. $$x_1 + 3x_2 - x_3 = 5$$ $$3x_1 - x_2 = 5$$ $$x_2 + 2x_3 = 1.$$ **OR** Using Gauss-Jacobi method, solve the following system of equations starting with initial solution as $x_1^{(0)} = \frac{9}{5}, x_2^{(0)} = \frac{4}{5}, x_3^{(0)} = \frac{6}{5}$. Perform three iterations. $$5x_1 - x_2 = 9$$ $$-x_1 + 5x_2 - x_3 = 4$$ $$-x_2 + 5x_3 = -6.$$	**[8]**	**CO4**
SECTION C **(Q10 is compulsory and Q11.A and Q11.B have internal choices)**			
10.A	Consider the system of equations $$x_1 + 6x_2 + 2x_3 = 9$$ $$2x_1 + 12x_2 + 5x_3 = -4$$ $$-x_1 - 3x_2 - x_3 = 17.$$ i. Show that the coefficient matrix $\begin{bmatrix} 1 & 6 & 2 \\ 2 & 12 & 5 \\ -1 & -3 & -1 \end{bmatrix}$ does not have an LU-decomposition.	**[10]**	**CO4**

	ii. Re-arrange the equations, and find the LU-decomposition of the new coefficient matrix. iii. Hence solve the system by LU-decomposition technique (Crout's method).		
10.B	Find the smallest positive root of the equation $10^x + x - 4 = 0$, by Newton-Raphson method, correct upto three decimal places.	**[10]**	**CO3**
11.A	Let a function $f(x)$ be known for $(n+1)$ distinct equi-spaced arguments, namely $x_0, x_1, x_2, \dots, x_n$ such that $x_r = x_0 + rh$, where h is the step length, and the corresponding arguments are given by $y_j = f(x_j)$, for $j = 0,1,2,\dots,n$. Derive Newton's forward interpolation formula to find an approximate polynomial $p(x)$ of degree less than equal to n which satisfies the conditions $p(x_j) = f(x_j) = y_j$, for $j = 0,1,2,\dots,n$. **OR** Let a function $f(x)$ be known for $(n+1)$ arguments, namely $x_0, x_1, x_2, \dots, x_n$, which are not necessarily equi-spaced, and $y_j = f(x_j)$, for $j = 0,1,2,\dots,n$ be the corresponding entries. Derive Newton's divided difference interpolation formula to construct an approximate polynomial $\phi(x)$ of degree less than equal to n which satisfies the conditions $\phi(x_j) = f(x_j) = y_j$, for $j = 0,1,2,\dots,n$.	**[10]**	**CO1**
11.B	Consider the following boundary value problem (BVP) $$\frac{d^2y}{dx^2} - y = x^2, 0 \le x \le 1$$ $$y(0) = 1, y(1) = 0.$$ Find an approximate solution $\bar{y}(x) = a_1\phi_1(x) + a_2\phi_2(x)$ by Galerkin's method. Consider the basis functions $\phi_1(x) = (1-x)$ and $\phi_2(x) = (1-x)^2$. **OR** Find an approximate solution of the following problem by Subdomain (Partition) method diving the interval $0 \le x \le 1$ two equal subintervals and using the basis functions $\phi_1(x) = (1-x)$ and $\phi_2(x) = (1-x)^2$. $$\frac{d^2y}{dx^2} - y = x, 0 \le x \le 1$$ $$y(0) = 1, y(1) = 0.$$	**[10]**	**CO6**