**UNIVERSITY OF PETROLEUM AND ENERGY STUDIES**
**End Semester Examination, April/May 2018**

**Course: Computational Fluid Dynamics** **Semester: VIII**
**Program: B. Tech. ASE, ASE+AVE**
**Time: 03 hrs.** **Max. Marks: 100**

**Instructions: The question paper has 03 pages.**

**SECTION A (5 x 4 =20 Marks)**

| S. No. | | Marks | CO |
|---|---|---|---|
| Q 1 | Write the full Navier-Stokes equation in strong conservation form. Elaborate the significance of each term. | | **CO1** |
| Q 2 | List down examples of flows governed by elliptic, parabolic, hyperbolic and mixed partial differential equations. | | **CO2** |
| Q 3 | Discuss the advantages and disadvantages of structured and unstructured grids. | | **CO3** |
| Q 4 | Explain the difference between finite difference and finite volume methods. | | **CO5** |
| Q 5 | Show that a proper choice of *weight function* makes the weighted residual formulation equivalent to Finite difference or Finite Volume Methods. | | **CO6** |

**SECTION B (4 x 10 = 40 Marks)**

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| Q 6 | Derive a second order accurate stencil for the mixed derivative $\frac{\partial^2 u}{\partial xdy}$. | | **CO3** |
| Q 7 | Explain the UPWIND interpolation scheme for the evaluation of fluxes at face centre using the nodal values on a structured finite volume grid and estimate its accuracy. Find an expression for the artificial diffusivity introduced by this scheme.<br><br>***OR***<br>Explain the CDS interpolation scheme for the evaluation of fluxes at face centre using the nodal values on a structured finite volume grid. Find the order of accuracy of this scheme. | | **CO5** |
| Q 8 | List down stepwise procedure for the solution of governing partial differential equation using Finite Element Discretization. | | **CO5** |
| Q 9 | Consider the 2-dimensional transient heat conduction equation given below. The Crank-Nicolson discretization of the equation results in a pentadiagonal system of equations. Describe the Alternating Direction Implicit (ADI) technique to solve the | | **CO6** |

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| | system of equations iteratively. $$\frac{\partial T}{\partial t} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$ | | |
| **SECTION-C (2 x 20 =40 Marks)** | | | |
| Q 10 | An explicit scheme for solving the first-order wave equation is given by: $$u_j^{n+1} = u_j^n - \frac{c\Delta t}{\Delta x}\left(u_j^n - u_{j-1}^n\right)$$ Apply the Fourier stability analysis to this scheme, and determine the stability restrictions, if any. <br> ***OR*** <br> Derive the *modified equation* that emanates from the first order forward in time and backward in space discretization of the first order wave equation given below. $$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0$$ Discuss the effect of dominating error for the above discretization and suggest means to minimize it. | | **CO4** |
| Q 11 | Consider the function $\phi(x, y) = e^x + e^y$. Consider the point $(x,y) = (1,1)$ <br> a. Calculate the exact values of $\frac{\partial \phi}{\partial x}$ and $\frac{\partial \phi}{\partial y}$ at this point. <br> b. Use first order forward differences, with $\Delta x = \Delta y = 0.1$, to calculate approximate values of $\frac{\partial \phi}{\partial x}$ and $\frac{\partial \phi}{\partial y}$ at point (1, 1). Calculate the percentage difference when compared with the exact values from part (a). <br> c. Use first order rearward differences with $\Delta x = \Delta y = 0.1$, to calculate approximate values of $\frac{\partial \phi}{\partial x}$ and $\frac{\partial \phi}{\partial y}$ at point (1, 1). Calculate the percentage difference when compared with the exact values from part | | **CO6** |

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| | | (a).<br>d. Use second order central differences, with $\Delta x = \Delta y = 0.1$, to calculate approximate values of $\frac{\partial \phi}{\partial x}$ and $\frac{\partial \phi}{\partial y}$ at point (1, 1). Calculate the percentage difference when compared with the exact values from part (a). | | |