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## ON SYNTHETIC AND COMPOSITE ESTIMATORS FOR SMALL AREA ESTIMATION UNDER LAHIRI — MIDZUNO SAMPLING SCHEME

G.C. Tikkiwal<sup>1</sup> and K. K. Pandey<sup>2</sup>

#### **ABSTRACT**

This paper studies performance of synthetic ratio estimator and composite estimator, which is a weighted sum of direct and synthetic ratio estimators, under Lahiri – Midzuno (L-M) sampling scheme. The synthetic estimator under L-M scheme is unbiased and consistent if the assumption of synthetic estimator is satisfied. Further, this paper compares performance of the synthetic and composite estimators empirically under L-M and SRSWOR schemes for estimating crop acreage for small domains. The study shows that both the estimators perform better under L-M scheme as having comparatively smaller absolute relative biases and relative standard errors.

**Key words**: Composite estimators, Synthetic ratio estimators, Small domains, Lahiri – Midzuno sampling design, SICURE model .

### 1. Introduction

Gonzalez and Wakesberg (1973) and Schaible, Brock, Casady and Schnack (1977) compare errors of synthetic and direct estimators for standard Metropolitan Statistical Areas and Counties of U.S.A. The authors of both the papers conclude that when in small domains sample sizes are relatively small the synthetic estimator out performs the simple direct, whereas, when sample sizes are large the direct outperforms the synthetic. These results suggest that a weighted sum of these two estimators, known as composite estimator, can provide an alternative to choosing one over the other. Tikkiwal, B.D. and Tikkiwal G.C. (1998) and Tikkiwal G.C. and Ghiya (2004) define a generalized class of composite estimators for small domains using auxiliary variable, under simple

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Deptt. of Mathematics & Statistics, J.N.V. University, Jodhpur-342 011, India; E-mail: gctikkiwal@yahoo.com

<sup>&</sup>lt;sup>2</sup> Banasthali Vidyapith, P.O. Box Banasthali Vidyapith - 304022, India

random sampling and stratified random sampling schemes. Further, the authors compare the relative performance of the estimators belonging to the generalized class with the corresponding direct and synthetic estimators. The study suggests the use of composite estimator, combining direct and synthetic ratio estimators, as it has smaller relative bias and standard error.

In this paper we study the performance of synthetic ratio estimator, and composite estimator belonging to the generalized class of composite estimators for small domains, under Lahiri-Midzuno scheme of sampling. The study suggests that the estimators perform better under Lahiri-Midzuno scheme of sampling than, under SRSWOR scheme.

#### 2. Notations

Suppose that a finite population  $U=(1,\ldots,i,\ldots,N)$  is divided into 'A' non overlapping small domains Ua of given size Na  $(a=1,\ldots,A)$  for which estimates are required. We denote the characteristic under study by 'y'. We further assume that the auxiliary information is available and denote this by 'x'. A random sample s of size n is selected through Lahri-Midzuno sampling scheme (1951, 52) from population U such that na units in the sample 's' comes from small domain Ua  $(a=1,\ldots,A)$ .

Consequently,

$$\sum_{a=1}^{A} N_a = N \quad and \quad \sum_{a=1}^{A} n_a = n$$

We denote the various population and sample means for characteristics Z = X, Y by

 $\overline{Z}$  = mean of the population based on N observations.

 $\overline{Z}a$  = population mean of domain 'a' based on Na observations.

 $\overline{z}$  = mean of the sample 's' based on n observations.

za = sample mean of domain 'a' based on na observations.

Also, the various mean squares and coefficient of variations of the population 'U' for characteristics Z are denoted by

$$S_z^2 = \frac{1}{N-1} \sum_{i=1}^N \left( z_i - \overline{Z} \right)^2, \quad C_z = \frac{S_z}{\overline{Z}}$$

The coefficient of covariance between X and Y is denoted by

$$C_{xy} = \frac{S_{xy}}{\overline{XY}}$$

where,

$$S_{xy} = \frac{1}{N-1} \sum_{i=1}^{N} \left( y_i - \overline{Y} \right) \left( x_i - \overline{X} \right)$$

The corresponding various mean squares and coefficient of variations of small domains Ua are denoted by

$$S_{z_a}^2 = \frac{1}{N_a - 1} \sum_{i=1}^{N_a} \left( Z_{a_i} - \overline{Z_a} \right)^2 \text{ , } C_{z_a} = \frac{S_{z_a}}{\overline{Z}_a} \quad \text{ and } C_{x_a y_a} = \frac{S_{x_a y_a}}{\overline{X}_a \overline{Y}_a}$$

where,

$$S_{x_a y_a} = \frac{1}{N_a - 1} \sum_{i=1}^{N_a} \left( y_{a_i} - \overline{Y}_a \right) \left( x_{a_i} - \overline{X}_a \right)$$

and zai (a = 1, ..., A and i = 1, ..., Na) denote the i-th observation of the small domain 'a' for the characteristic Z = X, Y.

## 3. Synthetic Ratio Estimator

We consider here synthetic ratio estimator of population mean  $\overline{Y}_a$ , based on auxiliary information 'x' under Lahiri-Midzuno sampling scheme, as described in previous section. The synthetic ratio estimator of population mean  $\overline{Y}_a$  of small area 'a' is defined as follows :

$$\overline{y}_{syn,a} = \frac{\overline{y}}{\overline{x}} \overline{X}_a \tag{3.1}$$

This estimator may be heavily biased unless the following assumption is satisfied

$$(\overline{Y}_a / \overline{X}_a) = (\overline{Y} / \overline{X}) \text{ for all } a \in A$$
 (3.2)

## 3.1.Bias and Mean Square Error

Under Lahiri-Midzuno sampling scheme

$$E\left(\overline{y}_{syn,a}\right) = E\left(\frac{\overline{y}}{\overline{x}} \overline{X}_{a}\right)$$

$$= \frac{\overline{X}_{a}}{\overline{X}} E\left(\frac{\overline{y}}{\overline{x}} \overline{X}\right)$$

$$= \frac{\overline{X}_{a}}{\overline{X}} \overline{Y}$$

$$= \frac{\overline{X}_{a}}{\overline{X}} \overline{Y}$$
(3.3)

Therefore, design bias of  $y_{syn,a}$  is

$$B\left(\overline{y}_{syn,a}\right) = \left(\frac{\overline{Y}}{\overline{X}} \overline{X}_a - \overline{Y}_a\right) = B_1(say)$$
(3.4)

The mean square error of  $y_{syn,a}$  is given by

$$MSE\left(\overline{y}_{syn,a}\right) = \frac{\overline{X}_{a}^{2}}{\overline{X}^{2}} V\left(\frac{\overline{y}}{\overline{x}}\overline{X}\right) + B_{1}^{2}$$

$$= \frac{\overline{X}_{a}^{2}}{\overline{X}} \left[\frac{1}{N}\sum_{c}\left(\frac{\overline{y}^{2}}{\overline{x}}\right)_{c} - \overline{Y}^{2}\right] + B_{1}^{2}$$
(3.5)

where,  $\frac{\sum_{c}}{stands}$  stands for summation over all possible samples.

#### Remark 3.1

The above expression of MSE  $(y_{syn,a})$  is not in analytical form.

#### Remark 3.2

If the synthetic assumption given in Eq. (3.2) satisfies then the  $B_1 = B(\overline{y}_{syn,a}) = 0$  and hence consistent estimator of MSE  $(\overline{y}_{syn,a})$  is given by

$$\operatorname{mse}(\overline{y}_{\operatorname{syn,a}}) = \frac{\overline{X}_{a}^{2}}{\overline{X}^{2}} \operatorname{v}(\overline{y}_{R})$$

$$= \frac{\overline{X}_{a}^{2}}{\overline{X}^{2}} \left[ \overline{y}_{R}^{2} - \frac{\overline{X}}{\overline{x}} \left\{ \overline{y}^{2} - \left( \frac{1}{n} - \frac{1}{N} \right) s_{y}^{2} \right\} \right]$$
(3.6)

where,  $\overline{y}_R = \frac{\overline{y}}{x} \overline{X}$ 

## 3.2 Comparison under SRSWOR

The expressions of Bias and Mean square error of synthetic ratio estimator under SRSWOR scheme is given by Tikkiwal & Ghiya (2000), while discussing the properties of generalized class of synthetic estimator, as under

$$B_2 = B\left(\overline{y}_{syn,a}\right) = \frac{\overline{Y}}{\overline{X}} \overline{X}_a \left[1 + \frac{N-n}{Nn} \left(C_x^2 - C_{xy}\right)\right] - \overline{Y}_a$$
(3.7)

and

$$MSE\left(\overline{Y}_{syn,a}\right) = \left(\frac{\overline{Y}}{\overline{X}}\overline{X}_{a}\right)^{2} \left[1 + \frac{N-n}{Nn}\left\{3C_{x}^{2} + C_{y}^{2} - 4C_{xy}\right\}\right]$$
$$-2\overline{Y}_{a}\left(\frac{\overline{Y}}{\overline{X}}\overline{X}_{a}\right) \left[1 + \frac{N-n}{Nn}\left(C_{x}^{2} - C_{xy}\right)\right] + \overline{Y}_{a}^{2}$$
(3.8)

Comparing the expression of biases B1 and B2 of  $y_{syn,a}$  under L-M & SRSWOR schemes, we get from Eqs. (3.4) and (3.7)

$$B_2 - B_1 = \frac{N - n}{Nn} \frac{\overline{Y}}{\overline{X}} \overline{X}_a \left( C_x^2 - C_{xy} \right)$$
(3.9)

So, 
$$B_2 \ge B_1$$
 if 
$$C_x^2 - C_{xy} \ge 0 \implies \rho \frac{C_y}{C_x} \le 1$$

#### Remark 3.3

If the synthetic assumption given in Eq. (3.2) satisfies then the expression of bias B2 given in Eq. (3.7) reduces to

$$B_2 = \frac{N - n}{Nn} \left( C_x^2 - C_{xy} \right) \tag{3.10}$$

That is,  $B2 \neq 0$  even if synthetic assumption is satisfied. Whereas under this condition B1 = 0.

#### Remark 3.4

If the synthetic assumption is satisfied than the expressions of MSE  $(y_{syn,a})$  given in Eq. (3.5) and Eq. (3.8) reduces respectively to

$$\mathbf{M}_{1} = \mathbf{MSE}\left(\overline{\mathbf{y}}_{\text{syn,a}}\right) = \frac{\overline{\mathbf{X}}_{a}^{2}}{\overline{\mathbf{X}}} \left[ \frac{1}{\left(\frac{\mathbf{N}}{\mathbf{n}}\right)} \sum_{c} \left(\frac{\overline{\mathbf{y}}^{2}}{\overline{\mathbf{x}}}\right)_{c} - \overline{\mathbf{Y}}^{2} \right]$$
(3.11)

and

$$M_2 = MSE(\overline{y}_{syn,a}) = \frac{N-n}{Nn} (C_x^2 + C_y^2 - 2 C_{xy})$$
 (3.12)

It may be noted here that the expression M1 under L-M design is still not in analytical form, therefore, a theoretical comparison of expressions M1 and M2 is not possible.

## 4. Composite Estimator

We consider in this section a composite estimator  $(\overline{y}_{c,a})$ , which is a combination of direct ratio  $(\overline{y}_{d,a})$  and synthetic ratio  $(\overline{y}_{syn,a})$  estimators, under L-M design.

That is,

$$\overline{y}_{c,a} = w_a \overline{y}_{d,a} + (1 - w_a) \overline{y}_{syn,a}$$

$$\overline{y}_{d,a} = \frac{\overline{y}_a}{\overline{x}_a} \overline{X}_a$$
Where, and wa is suitably chosen constant.

It may noted that the estimator  $y_{d,a}$  is design biased whereas  $y_{sym,a}$  is design unbiased estimator under L-M design. When domain size Na is known, an estimator of the approximate variance of  $y_{d,a}$  under L-M design is given by

$$\hat{V}(\bar{y}_{d,a}) = \left\{ \frac{\bar{X}_a}{\sum_{i \in s_a} (x_i / \pi_i)} \right\}^2 \sum_{i,j \in s_a} \Delta_{ij} \left( \frac{y_i - \hat{B}_a x_i}{\pi_i} \right) \left( \frac{y_j - \hat{B}_a x_j}{\pi_j} \right) \tag{4.2}$$

Where, sa =  $Ua \cap s$ . That is, sa is the subset of sample s that falls in the domain Ua.

$$\begin{split} & \Delta_{ij} = \frac{\pi_{ij} - \pi_{i}\pi_{j}}{\pi_{ij}} \quad \hat{B}_{a} = \underbrace{\sum_{i \in s_{a}} (y_{i}/\pi_{i})}_{\sum_{i \in s_{a}} (x_{i}/\pi_{i})}, \\ & \pi_{i} = \frac{N - n}{N - 1}p_{i} + \frac{n - 1}{N - 1} \\ & \pi_{ij} = \left\{ \frac{\binom{n - 1}{N - 1}}{\binom{N - n}{N - 2}} (\binom{N - n}{N - 2})(p_{i} + p_{j}) + \binom{n - 2}{N - 2} \right\} \text{ for all } i \neq j \\ & \frac{N - n}{N - 1}p_{i} + \frac{n - 1}{N - 1} \quad \text{ for all } i = j \end{split}$$

and 
$$p_i = x_i / X$$
  
[cf. Sarandal et al. (1992) Eq. (10.6.3)]

## 4.1 Estimation of Weights

The optimum values  $\overset{\mathbf{W}_{a}^{'}}{\overset{}{\mathbf{y}}}$  of wa may be obtained by minimizing the mean square error of  $\overset{\mathbf{V}_{c,a}}{\overset{}{\mathbf{y}}}$  with respect to wa and it is given by

$$w_{a}^{'} = \frac{MSE(\bar{y}_{syn,a}) - E(\bar{y}_{d,a} - \bar{Y}_{a})(\bar{y}_{syn,a} - \bar{Y}_{a})}{MSE(\bar{y}_{d,a}) + MSE(\bar{y}_{syn,a}) - 2E(\bar{y}_{d,a} - \bar{Y}_{a})(\bar{y}_{syn,a} - \bar{Y}_{a})}$$

Under the assumption that  $E(\overline{y}_{d,a}-\overline{Y}_a)(\overline{y}_{syn,a}-\overline{Y}_a)$  is small relative to  $MSE(\overline{y}_{syn,a})$ , the  $W_a$  reduced to

$$w_{a}^{*} = \frac{MSE(\bar{y}_{syn,a})}{MSE(\bar{y}_{d,a}) + MSE(\bar{y}_{syn,a})}$$
(4.3)

Since  $y_{\text{syn,a}}$  is not an unbiased estimator, therefore, an unbiased estimator of MSE( $y_{\text{syn,a}}$ ) under the assumption that  $Cov(y_{\text{d,a}}, y_{\text{syn,a}}) = 0$ , is given by [cf. Rao (2003), Eq. 4.2.12)]

$$mse(\overline{y}_{syn,a}) = (\overline{y}_{syn,a} - \overline{y}_{d,a}) 2 - v(\overline{y}_{d,a})$$
(4.4)

Now, using Eq. (3.6) the weights  $W_a^*$  can be estimated as follows:

$$\hat{w}_{a}^{*} = \frac{mse(\bar{y}_{syn,a})}{(\bar{y}_{syn,a} - \bar{y}_{d,a})^{2}}$$
(4.5)

But this estimator of  $W_a^*$  can be very unstable. Schaible (1978) proposes an average weighting scheme based on several variables or "similar" areas or both, to overcome this difficulty. In our empirical study presented in next section, we take average of  $\hat{W}_a^*$  over "similar" areas.

# 5. Crop Acreage Estimation for Small Domains — A Simulation Study

In this section we compare the relative performance of  $y_{syn,a}$  and  $y_{c,a}$  under L-M and SRSWOR sampling schemes, through a simulation study, as the mean square errors of  $y_{d,a}$  and  $y_{syn,a}$ , and hence of  $y_{c,a}$  are not in analytical form. This we do by taking up the State of Rajasthan, one of the states in India, for our case study.

#### 5.1 Existing methodology for estimation

In order to improve timelines and quality of crop acreage statistics, a scheme known as Timely Reporting Scheme (TRS) has been in vogue since early seventies in most of the States of India. The TRS has the objective of providing quick and reliable estimates of crop acreage statistics and there-by production of the principle crops during each agricultural season. Under the scheme the Patwari (Village Accountant) is required to collect acreage statistics on a priority basis in a 20 percent sample of villages, selected by stratified linear systematic sampling design taking Tehsil (a sub-division of the District) as a stratum. These statistics are further used to provide state level estimates using direct estimators viz. Unbiased (based on sample mean) and ratio estimators.

The performance of both the estimators in the State of Rajasthan, like in other states, is satisfactory at state level, as the sampling error is within 5 percent. However, the sampling error of both the estimators increases considerably, when they are used for estimating acreage statistics of various principle crops even at district level, what to speak of levels lower than a district. For example, the sampling error of direct ratio estimator for Kharif crops (the crop sown in June-July and harvested in October- November every year) of Jodhpur district (of Rajasthan State) for the agricultural season 1991-92 varies approximately between 6 to 68 percent. Therefore, there is need to use indirect estimators at district and lower levels for decentralized planning and other purposes like crop insurance, bank loan to farmers.

## 5.2 Details of the simulation study

For collection of revenue and administrative purposes, the State of Rajasthan, like most of the other states of India, is divided into a number of districts.

Further, each district is divided into a number of Tehsils and each Tehsil is also divided into a number of Inspector Land Revenue Circles (ILRCs). Each

ILRC consists of a number of villages. For the present study, we take ILRCs as small domains.

In the simulation study, we undertake the problem of crop acreage estimation for all Inspector Land Revenue Circles (ILRCs) of Jodhpur Tehsil of Rajasthan. They are seven in number and these ILRCs contain respectively 29, 44, 32, 30, 33, 40 and 44 villages. These ILRCs are small domains from the TRS point of view. The crop under consideration is Bajra (Indian corn or millet) for the agriculture season 1993-94. The bajra crop acreage for agriculture season 1992-93 is taken as the auxiliary characteristic x.

We consider the following estimators of population total Ta of small domain 'a' for a = 1,2,...,7

Synthetic ratio estimator 
$$t_{1,a} = N_a \left(\frac{\overline{y}}{\overline{x}}\right) \overline{X}_a$$

and

Composite estimator  $t2,a = Na \ Y_{c,a}$ 

To assess the relative performance of the estimators under two different sampling schemes viz. L-M and SRSWOR, their Absolute Relative Bias (ARB) and Simulated relative standard error (Srse) are calculated for each ILRC as follows:

$$ARB(t_{k,a}) = \frac{\left| \frac{1}{500} \sum_{s=1}^{500} t_{k,a}^{s} - T_{a} \right|}{T_{a}} x100$$
 (5.2.1)

and

Srse(
$$t_{k,a}$$
) =  $\frac{\sqrt{\text{SMSE}(t_{k,a})}}{T_a}$  x100 (5.2.2)

where

SMSE(
$$t_{k,a}$$
) =  $\frac{1}{500} \sum_{s=1}^{500} (t_{k,a}^s - T_a)^2$  (5.2.3)

for k = 1, 2 and a = 1, ..., 7

#### 5.3 Results

We present the results of ARB (in %) synthetic ratio estimator  $(\overline{y}_{syn,a})$  in Table 5.3.2 and of composite estimator  $(\overline{y}_{c,a})$  in Table 5.3.3. The Srse (in %) of composite estimator are presented in Table 5.3.4 and Table 5.3.5. The total number of villages in Jodhpur Tehsil is 252. We take n = 25, 50, 63 and 76 i.e. samples, approximately, of 10%, 20%, 25% and 30% villages. It may be noted

that a sample of 20% villages are presently adopted in TRS. Before simulation, we first examined the validity of synthetic assumption given in Eq. (3.1) . The results of these are presented in Table 5.3.1. From this we note that the assumption closely meets for ILRCs (3), (5) and (7) . Where as, the assumption deviate moderately for ILRC (4) , and deviate considerably for ILRCs (1) and (2). In case of composite estimators, we estimate the weights for each small domain using Eq. (4.5) but for estimating total of small domains of ILRCs (3), (5) and (7)

we take average of  $\hat{W}_a^*$  over these domains, being "similar".

We observe from Table 5.3.2 to Table 5.3.5 (specially for n=50 i.e. a sample of 20% villages that is being selected under TRS scheme) that both the estimators perform well in ILRCs (3), (5) and (7) under both the sampling schemes, where synthetic assumption closely satisfied . But the composite estimator  $(y_{c,a})$  performs better than the synthetic ratio estimator. The ARB of both the estimators under consideration is much smaller in case of L-M scheme than in case of SRSWOR. Also the Srse of both the estimators reduces under L-M scheme and is about 5%. Here we suggest that when the synthetic assumption is not valid one should look for other types of estimators such as those obtained through the SICURE MODEL [B.D.Tikkiwal (1993)] or presented in Ghosh and Rao (1994).

**Table 5.3.1** Absolute Differences (Relative) under Synthetic Assumption of Synthetic Ratio Estimator for Various ILRCs.

ILRC	$\overline{Y}_a$ / $\overline{X}_a$	$\overline{\overline{Y}}/\overline{\overline{X}}$	$\left[\left \overline{Y}_{a}/\overline{X}_{a}-\overline{Y}/\overline{X}\right /\left(\overline{Y}_{a}/\overline{X}_{a}\right)\right]X100$
(1)	.7303	.8675	18.17
(2)	.7402	.8675	17.19
(3)	.8663	.8675	0.13
(4)	.9416	.8675	7.86
(5)	.8595	.8675	0.91
(6)	.9666	.8675	10.25
(7)	.8815	.8675	1.58

Source: own calculations.

**Table 5.3.2** Absolute Relative Biases (in %) of Synthetic Ratio Estimator under L-M and SRSWOR Designs for different sample sizes.

ILRC	For n = 25		For n = 50		For $n = 63$		For n = 76	
ILKC	LM	SRSWOR	LM	SRSWOR	LM	SRSWOR	LM	SRSWOR
(1)	17.06	18.01	15.88	17.90	14.01	17.68	13.65	18.02
(2)	18.79	19.65	9.01	19.5	8.94	19.32	7.05	19.66
(3)	0.59	0.62	0.016	0.72	0.011	0.895	0.008	0.61
(4)	1.06	8.57	1.28	8.66	1.13	8.81	1.11	8.55
(5)	0.132	0.156	0.021	0.55	0.014	0.11	0.012	0.17
(6)	8.34	10.94	7.79	11.03	5.83	11.18	5.14	10.93
(7)	0.96	1.12	0.34	1.02	0.26	0.85	0.22	1.13

Source: own calculations.

**Table 5.3.3** Absolute Relative Biases (in %) of Composite Estimator under L-M and SRSWOR Designs for different sample sizes.

ILRC	For n = 25		For $n = 50$		For $n = 63$		For n = 76	
	LM	SRSWOR	LM	SRSWOR	LM	SRSWOR	LM	SRSWOR
(1)	9.68	10.72	8.10	8.40	7.65	8.01	4.63	5.18
(2)	11.53	12.6	8.76	10.02	5.43	7.60	5.15	6.42
(3)	0.36	1.98	0.009	0.50	0.006	0.53	.008	0.28
(4)	6.97	7.57	1.19	6.30	2.19	5.20	2.08	4.73
(5)	0.105	0.01	0.019	0.38	0.008	0.29	0.007	0.41
(6)	7.14	7.60	3.45	4.60	4.19	4.60	3.01	3.51
(7)	0.83	1.53	0.24	1.20	0.18	1.01	0.17	1.40

Source: own calculations.

**Table 5.3.4** Simulated Relative Standard Error (Srse in %) of Synthetic Ratio Estimator under L-M and SRSWOR Designs for different sample sizes.

ILRC	For n = 25		For n = 50		For n = 63		For n = 76	
ILKC	LM	SRSWOR	LM	SRSWOR	LM	SRSWOR	LM	SRSWOR
(1)	19.87	20.15	18.34	19.11	17.67	18.07	19.78	18.67
(2)	21.34	22.34	19.39	20.67	19.81	20.01	18.54	19.98
(3)	7.15	7.67	5.01	5.71	5.03	5.15	5.51	5.01
(4)	10.13	11.08	9.87	10.10	9.81	10.01	8.31	9.87
(5)	7.65	8.14	5.14	5.91	5.01	5.05	4.98	5.01
(6)	16.01	15.13	11.13	12.14	12.15	13.14	11.98	13.06
(7)	6.85	7.97	5.36	5.85	4.98	5.18	5.11	5.08

Source: own calculations.

**Table 5.3.5** Simulated Relative Standard Error (Srse in %) of Composite Estimator under L-M and SRSWOR Designs for different sample sizes.

ILRC	For n = 25		For $n = 50$		For $n = 63$		For $n = 76$	
ILKC	LM	SRSWOR	LM	SRSWOR	LM	SRSWOR	LM	SRSWOR
(1)	17.65	18.93	13.67	16.48	14.65	15.83	15.01	16.71
(2)	14.98	15.61	11.81	13.48	12.74	13.01	11.82	14.63
(3)	6.08	6.81	4.34	4.78	4.11	4.54	4.08	4.89
(4)	11.98	12.34	9.16	10.15	8.84	9.71	8.01	8.76
(5)	6.34	6.98	4.73	5.01	4.25	4.98	4.13	4.31
(6)	9.24	9.89	7.63	8.13	8.01	7.63	6.79	7.01
(7)	7.11	7.63	5.14	5.44	4.91	5.31	4.16	5.28

Source: own calculations.

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#### REFERENCE

- GHOSH, M. and RAO, J.N.K. (1994). Small Area Estimation: An Appraisal. Statistical Science, 91, 55—93.
- GONZALEZ, N.E. and WAKSBERG, J.(1973). Estimation of the error of synthetic estimates. Paper presented at first meeting of international association of survey statisticians, Vienna, Austria, 18—25.
- LAHIRI, D.B. (1951). A method of sample selection providing unbiased ratio estimates. Bull. Int. Stat. Inst. 3, 133—40.
- MIDZUNO, H. (1952). On the sampling system with probability proportional to sum of sizes, Ann. Inst. Stat. Math., 3, 99—107.
- Rao, J. N. K. (2003). Small area estimation. Wiley Interscience
- Sarndal, C. E., Swensson, B. and Wretman, J. (1992). Model assisted survey sampling, Springer Verlag.
- SCHAIBLE, W.L. (1978). Choosing weights for composite estimators for small area statistics. Proceedings of the survey research methods section, Amer. Statist. Assoc., Washington. D.C., 741—746.
- SCHAIBLE, W.L., BROCK, D.B., CASADY, R.J. and SCHNACK, G.A. (1977). An empirical comparison of the simple synthetic and composite estimators for small area statistics. Proceedings of Amer. Statist. Assoc., Social Statistics section 1017—1021.
- TIKKIWAL, B.D. (1993). Modeling through survey data for small domains. Proceedings of International Scientific Conference on small Area Statistics and Survey Design (An invited paper), held in September 1992 at Warsaw, Poland.
- TIKKIWAL, G.C. and GHIYA A. (2000). A generalized class of synthetic estimators with application of crop acreage estimation for small domains. Biom, J. 42, 7, 865—876.
- TIKKIWAL, G.C. and GHIYA A. (2004). A generalized class of composite estimators with application to crop acreage estimation for small domains. Statistics in Transitions (6), 5, 697—711.